Robust Beamforming in Cognitive Radio

Gan Zheng, *Member, IEEE,* Shaodan Ma, *Member, IEEE,* Kai-Kit Wong, *Senior Member, IEEE,*
and Tung-Sang Ng, *Fellow, IEEE*

Abstract—This letter considers the multi-antenna cognitive radio (CR) network, which has a single secondary user (SU) and coexists with a primary network of multiple users. Our objective is to maximize the service probability of the SU, subject to the interference constraints on the primary users (PUs) in the form of probability. Exploiting imperfect channel state information (CSI), with its error modeled by added Gaussian noise, we address the optimization for the beamforming weights at the secondary transmitter. In particular, this letter devises an iterative algorithm that can efficiently obtain the robust optimal beamforming solution. For the case with one PU, we show that a much simpler algorithm based on a closed-form solution for the antenna weights of a given power can be presented. Numerical results reveal that the optimal solution for the constructed problem provides an effective means to tradeoff the performance between the PUs and the SU, bridging the non-robust and worst-case based systems.

Index Terms—Cognitive radio, interference control, robust beamforming.

I. INTRODUCTION

RADIO spectrum is a precious resource for wireless communications. According to federal communications commission (FCC) [1], spectrum utilization depends very much upon place and time and yet most spectrum is under-utilized. Cognitive radio (CR), first proposed by Mitola and Maguire in 1999 [2], is a new paradigm for exploiting the spectrum resources in a dynamic way [3], [4] and has been adopted in IEEE 802.22 Wireless Regional Area Networks (WRANs) for license-exempt devices to use the spectrum on a non-interfering basis.

Spectrum holes are the most obvious opportunities to be exploited by CR [5], but higher spectrum utilization is anticipated if coexistence between the primary (PU) and secondary users (SUs) is permitted. The latter is possible if the interference caused by the SUs can be properly controlled [6]. In this respect, multi-antenna beamforming has been recognized as an effective means to mitigate co-channel interference and widely used in traditional fixed-spectrum-allocation based wireless communications systems. However, the use of beamforming for interference control in CR is much more challenging

G. Zheng and K. K. Wong are with the Department of Electronic and Electrical Engineering, University College London, WC1E 7JE, UK (e-mail: {g.zheng, kwong}@ee.ucl.ac.uk).

S. Ma and T. S. Ng are with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong (e-mail: {sdma, tsng}@eee.hku.hk).

because the interference to the PUs from the SUs has to be kept below a limit.

In the literature, beamforming techniques have been devised for the secondary CR network to control interference and also achieve various objectives, such as capacity maximization [7], signal-to-interference plus noise ratio (SINR) balancing [7], and transmit power minimization with SUs' quality-of-service (QoS) constraints [8]. To summarize, most were largely based on the assumption of perfect channel state information (CSI) at the SU transmitter and the SU receiver, as well as the PU receivers, which is usually difficult to achieve due to limited training, less cooperation between SU and PU, or other factors such as quantization. Most recently in [9], given perfect CSI between the SU transmitter and receiver and imperfect CSI between the SU transmitter and the PU receiver, the beamforming design for a secondary CR user coexisting with a single PU was addressed.

In this letter, we consider a more general setting where there are multiple PUs coexisting with a SU and optimize the transmit beamforming at the secondary CR network for interference control with the aid of imperfect CSI at the SU transmitter, with its error modeled as additive Gaussian noise. Our problem is related to robust optimization against channel mismatches, which is usually tackled by either worst-case optimization [10] or stochastic analysis [11]. For the case when the CSI error is unbounded, for instance, due to imperfect estimation from training, statistical methods are more suitable and robustness is achieved in the form of confidence level measured by probability.

This letter aims to maximize the service probability of the SU while controlling the interference levels to the PUs based on some preset probability constraints by optimizing the beamforming at the SU transmitter in accordance with imperfect CSI. The construction of the problem facilitates a soft tradeoff on the performance between the PUs and the SU, offering an analytical connection between a selfish non-robust secondary system and the conservative (sometimes unachievable) worst-case robust SU solution. We show that the optimal robust beamforming solution for the general problem can be obtained. For the special case with only one PU, a much simpler analytical method, which is based on a closed-form solution for the antenna weights of a given transmit power, is devised.

In the sequel, we use the following notations. Vectors are in columns and denoted by lowercase bold letters, while matrices are denoted by uppercase bold letters. The superscripts, † and ^{T}, denote the conjugate transposition and the transposition, respectively. Also, $|\cdot|$ takes the modulus of a complex number and $\|\cdot\|$ returns the Frobenius norm, while $\text{Im}\{\cdot\}$ outputs the

imaginary part of an input number. The real number field is denoted by $\mathbb{R}$. The notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{V})$ states that $\mathbf{x}$ contains entries of complex Gaussian random variables, with mean $\mathbf{m}$ and covariance $\mathbf{V}$.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a CR network with $L (\geq 1)$ PUs and one SU. The SU transmitter has N antennas while there is only one antenna at the SU receiver and at each of the PUs. The channels between the SU transmitter and the PUs are denoted by $\{\mathbf{g}_l\}$ for $l = 1, \ldots, L$ and we use $\mathbf{h}$ to denote the channel between the SU transmitter and receiver. Our problem is to maximize the SU's received power for a given transmit power constraint P while controlling the interferences on the PUs to certain acceptable levels, say $\{I_l\}$. With a beamforming vector $\mathbf{w}$ at the SU transmitter, we have

$$\max_{\|\mathbf{w}\|^2 \leq P} |\mathbf{h}^\dagger \mathbf{w}|^2 \quad \text{s.t.} \quad |\mathbf{g}_l^\dagger \mathbf{w}|^2 \leq I_l, \forall l. \tag{1}$$

While in practice, the CSI available to the SU transmitter is destined to be imperfect, due to estimation errors or other factors such as quantization. In particular, in this letter, we model these errors as additive complex Gaussian noise so that

$$\begin{cases} \mathbf{h} = \hat{\mathbf{h}} + \Delta \mathbf{h} \\ \mathbf{g}_l = \hat{\mathbf{g}}_l + \Delta \mathbf{g}_l, \forall l, \end{cases} \tag{2}$$

where $\hat{\mathbf{h}}$ and $\{\hat{\mathbf{g}}_l\}$ denote the channel estimates known at the SU transmitter, and $\Delta \mathbf{h}$ and $\{\Delta \mathbf{g}_l\}$ are the respective CSI errors, which are specifically modeled as [11]

$$\begin{cases} \Delta \mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \sigma_h^2 \mathbf{I}), \\ \Delta \mathbf{g}_l \sim \mathcal{CN}(\mathbf{0}, \sigma_l^2 \mathbf{I}), \forall l, \end{cases} \tag{3}$$

with the variances σ_h^2 and $\{\sigma_l^2\}$ indicating the CSI quality.

Given this model, the optimization problem becomes

$$\max_{\|\mathbf{w}\|^2 \leq P} \text{Prob}\left(|\mathbf{h}^\dagger \mathbf{w}|^2 \geq \gamma\right) \quad \text{s.t.} \quad \text{Prob}\left(|\mathbf{g}_l^\dagger \mathbf{w}|^2 \leq I_l\right) \geq \varepsilon_l, \forall l, \tag{4}$$

where the probabilistic measures are done over the CSI error statistics. Note that the optimization is performed to maximize the service probability of the SU defined at a given target signal power threshold, γ, and the interferences are controlled probabilistically at some predetermined levels, $\{\varepsilon_l\}$, which can be chosen carefully to softly tradeoff the performance between the PUs and the SU.

To proceed, we express the service probability by noting that

$$y \triangleq |\mathbf{h}^\dagger \mathbf{w}|^2 = |\hat{\mathbf{h}}^\dagger \mathbf{w} + \Delta \mathbf{h}^\dagger \mathbf{w}|^2, \tag{5}$$

which is recognized as a non-central Chi-square random variable with degrees of freedom $n = 2$, variance $\sigma_y^2 = \frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}$ and noncentrality parameter $s_y^2 = |\hat{\mathbf{h}}^\dagger \mathbf{w}|^2$. As such,

$$\text{Prob}\left(|\mathbf{h}^\dagger \mathbf{w}|^2 \geq \gamma\right) = Q\left(\frac{s_y}{\sigma_y}, \frac{\sqrt{\gamma}}{\sigma_y}\right), \tag{6}$$

where $Q(, \cdot)$ denotes the generalized Marcum's Q-function [12, eq. (2.1–122)]. Moreover, we define two useful inverse functions, Q_1^{-1} and Q_2^{-1}, with regard to the first (or second)

parameter given the second (or first) parameter and the probability, respectively. That is, if $Q(a, b) = \xi$, then we have

$$\begin{cases} Q\left(Q_1^{-1}(b, \xi), b\right) = \xi, \\ Q\left(a, Q_2^{-1}(a, \xi)\right) = \xi. \end{cases} \tag{7}$$

Before proceeding, we state some useful properties of $Q(\cdot, \cdot)$ as follows.

P1. The generalized Marcum's Q-function, $Q(a, b)$, is non-decreasing with respect to a and non-increasing with respect to b.

P2. Given the probability ξ, $Q_1^{-1}(b, \xi)$ and $Q_2^{-1}(a, \xi)$ are both non-decreasing functions with respect to a and b, respectively.

Similarly, we can also express the interference probability constraints in the generalized Marcum's Q-function, $Q(\cdot, \cdot)$. As a consequence, (4) can be rewritten as follows:

$$\max_{\|\mathbf{w}\|^2 \leq P} \quad Q\left(\frac{|\hat{\mathbf{h}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}, \frac{\sqrt{\gamma}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}\right) \tag{8}$$

$$\text{s.t.} \quad Q\left(\frac{|\hat{\mathbf{g}}_l^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}}, \frac{\sqrt{I_l}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}}\right) \leq 1 - \varepsilon_l, \forall l.$$

The rest of the letter will be devoted to finding the optimal solution of (8).

III. OPTIMAL ROBUST BEAMFORMING IN CR

To solve (8), we observe that in both the objective function and the constraints the design variable $\mathbf{w}$ is involved in each parameter of the complicated generalized Marcum's Q-function. Due to the interference constraints, in general, it is anticipated that the SU's transmit power will not reach its maximum limit, P, and this is one of the reasons that makes it difficult to deal with. A closer observation reveals that the signal power $\|\mathbf{w}\|^2$ can be treated as a single parameter that influences the system performance. In what follows, we rewrite (8) as

$$\max_{\|\mathbf{w}\|^2 \leq P} Q\left(\frac{|\hat{\mathbf{h}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}, \frac{\sqrt{\gamma}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}\right) \tag{9}$$

$$\text{s.t.} \quad |\hat{\mathbf{g}}_l^\dagger \mathbf{w}| \leq Q_1^{-1}\left(\frac{\sqrt{I_l}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}}, 1 - \varepsilon_l\right)\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}, \forall l.$$

The above reformulation has inspired us to solve (8) by first addressing the problem for a given transmit power $\|\mathbf{w}\|^2 = p$, for some $p \leq P$, and then searching for the optimal p.

A. Optimal $\mathbf{w}$ for a Fixed Given p

To tackle (9) for a given power $\|\mathbf{w}\|^2 = p$, we need to solve the following problem

$$\max_{\|\mathbf{w}\|^2 = p} |\hat{\mathbf{h}}^\dagger \mathbf{w}| \quad \text{s.t.} \quad |\hat{\mathbf{g}}_l^\dagger \mathbf{w}|^2 \leq I_l', \forall l, \tag{10}$$

where $I_l' \triangleq Q_1^{-1}\left(\frac{\sqrt{I_l}}{\sqrt{\frac{p \sigma_l^2}{2}}}, 1 - \varepsilon_l\right)\sqrt{\frac{p \sigma_l^2}{2}}$.

The above objective function can be replaced by $\hat{\mathbf{h}}^\dagger \mathbf{w}$ without loss of optimality if $\mathrm{Im}\{\hat{\mathbf{h}}^\dagger \mathbf{w}\}$ is made zero. The equality constraint $\|\mathbf{w}\|^2 = p$, however, makes (10) nonconvex, and is difficult to handle. For this reason, we propose to solve the relaxed second-order cone-programming (SOCP) problem:

$$\max_{\|\mathbf{w}\|^2 \leq p} \hat{\mathbf{h}}^\dagger \mathbf{w} \quad \text{s.t.} \quad \begin{cases} \mathrm{Im}\{\hat{\mathbf{h}}^\dagger \mathbf{w}\} = 0, \\ |\hat{\mathbf{g}}_l^\dagger \mathbf{w}|^2 \leq I_l', \forall l. \end{cases} \tag{11}$$

The main advantage of this formulation is that (11) is now convex and standard interior point algorithms can be used to efficiently and optimally solve it. The rationale behind is that if there exists some optimal p and if it is known to (11), then the resulting $\mathbf{w}$ must satisfy the equality $\|\mathbf{w}\|^2 = p$. This means that at the optimum, the relaxation is tight, and therefore (11) is useful to derive the exact optimal solution to the original problem (8).

B. Optimum by SOCP and One-Dimensional Exhaustive Search

For a given p, if the obtained beamforming vector satisfies $\|\mathbf{w}\|^2 \leq p$, while the terms $\|\mathbf{w}\|^2$ in the objective function and constraints are taken as p for some known $p \leq P$. An important link for this to the original problem (9) is that if the optimal p, denoted by p_{opt}, is known and input to the relaxation (11), then the optimal $\mathbf{w}$ obtained from (11) must satisfy $\|\mathbf{w}\|^2 = p_{\mathrm{opt}}$ and thus gives the overall optimal solution for (9). In other words, very importantly, at the optimum state, this relaxation is tight. More importantly, this property provides a necessary condition for the optimal beamforming solution to be identified. In particular, if we solve (11) for any $0 < p \leq P$, we can identify all the possible solutions of $\mathbf{w}$ in (11) such that $\|\mathbf{w}\|^2 = p$ and among them, the one that maximizes the objective function of (9) gives the optimal robust beamforming solution for (9). In other words, it is thus possible to optimally solve (9) by repeatedly solving the SOCP (11) in combination with a one-dimensional search over $0 < p \leq P$ (see Algorithm 1).

Algorithm 1 Robust Beamforming by SOCP with One-Dimensional Search

1: **Input:** $\hat{\mathbf{h}}$, $\{\hat{\mathbf{g}}_l\}_{l=1}^L$, σ_h^2, $\{\sigma_l^2\}$, ΔP, and P.
2: **begin**
3:     Initialize the index $i = 1$, the set $\mathcal{S} = \emptyset$ and $p = \Delta P$.
4:     **if** $p \leq P$, **then**
5:         Solve (11) using SOCP.
6:         **if** $\|\mathbf{w}\|^2 = p$, **then**
7:             Store this solution to the set $\mathcal{S}$, i.e., $\mathcal{S} := \mathcal{S} \bigcup \{\mathbf{w}\}$.
8:         **end**
9:         Update $i := i + 1$ and $p := i\Delta P$.
10:    **end**
11:    Solve $\mathbf{w}_{\mathrm{opt}} = \arg\max_{\mathbf{w} \in \mathcal{S}} Q\left(\frac{|\hat{\mathbf{h}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}, \frac{\sqrt{\gamma}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}} \right)$.
12: **end**
13: **Output:** $\mathbf{w}_{\mathrm{opt}}$.

IV. THE SPECIAL CASE: $L = 1$

We have already addressed the optimization of robust beamforming for the general case of L PUs. In this section, we look at the special case when $L = 1$ or there is only one PU in the network. In this case, we shall show that an analytical solution for the optimal robust beamforming is possible, which helps reduce the required complexity for optimization significantly.

When $L = 1$, there is only one interference constraint and (8) becomes

$$\max_{\mathbf{w}} \quad Q\left(\frac{|\hat{\mathbf{h}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}}, \frac{\gamma}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_h^2}{2}}} \right) \tag{12}$$

$$\text{s.t.} \quad \begin{cases} \|\mathbf{w}\|^2 \leq P, \\ Q\left(\frac{|\hat{\mathbf{g}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}}, \frac{\sqrt{I}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}} \right) \leq 1 - \varepsilon. \end{cases}$$

The following corollary states a key fact at the optimum state.

Corollary 1. *At least one inequality constraint in (12) becomes active at the optimum.*

This result is rather obvious. If none of the constraints is active, then the SU's transmit power can always be increased to improve its service probability until one of the constraints becomes active. Now, let us discuss the solution if one of the constraints is active as follows.

C1: If $\|\mathbf{w}\|^2 = P$ and the interference constraint is not active, then the optimal transmit power is P and the interference constraint can be dropped, with the optimal $\mathbf{w}_{\mathrm{opt}}$ as

$$\mathbf{w}_{\mathrm{opt}} = \sqrt{P} \frac{\hat{\mathbf{h}}^\dagger}{\|\hat{\mathbf{h}}\|}. \tag{13}$$

The optimality of P and $\mathbf{w}_{\mathrm{opt}}$ in (13) can be easily detected by substituting (13) into the interference constraint. If it is satisfied, then P and (13) are indeed optimal.

C2: The interference constraint is active regardless of whether $\|\mathbf{w}\|^2 = P$. The optimal robust beamforming solution in this case is less obvious, and is addressed through the geometrical understanding of the problem structure described in the remainder of this section.

A. Upper and Lower Bounds on $\|\mathbf{w}\|^2 = p$

The transmit power of the SU can vary in the interval $p \in (0, P]$, and the higher the transmit power, the less the likelihood of the interference requirement being met. In this subsection, we present the upper and lower bounds on p, provided the interference constraint is active.

Lemma 1. *The allowable transmit power, $\|\mathbf{w}\|^2 = p$, satisfies $\underline{P} \leq p \leq \bar{P}$, where the bounds are given, respectively, by*

$$\underline{P} = \frac{2I}{\sigma^2 \left[Q_2^{-1} \left(\frac{\sqrt{2}\|\hat{\mathbf{g}}\|}{\sigma}, 1 - \varepsilon \right) \right]^2}, \tag{14a}$$

$$\bar{P} = \frac{2I}{\sigma^2 \left[Q_2^{-1} (0, 1 - \varepsilon) \right]^2}. \tag{14b}$$

Proof: From the interference constraint in (12), it can be seen that the first parameter of Q depends only on the direction, $\boldsymbol{v} \triangleq \frac{\mathbf{w}}{\|\mathbf{w}\|}$, while the secondary parameter depends only on the power level $\|\mathbf{w}\|^2 = p$. Satisfying the interference constraint in (12) with equality, the second parameter can be expressed as

$$\frac{\sqrt{I}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}} = Q_2^{-1}\left(\frac{|\hat{\mathbf{g}}^\dagger \mathbf{w}|}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}}, 1-\varepsilon\right)$$

$$= Q_2^{-1}\left(\frac{\sqrt{2}|\hat{\mathbf{g}}^\dagger \boldsymbol{v}|}{\sigma}, 1-\varepsilon\right), \qquad (15)$$

and the power in this case can be found as

$$p = \|\mathbf{w}\|^2 = \frac{2I}{\sigma^2\left[Q_2^{-1}\left(\frac{\sqrt{2}|\hat{\mathbf{g}}^\dagger \boldsymbol{v}|}{\sigma}, 1-\varepsilon\right)\right]^2}. \qquad (16)$$

Due to the monotonicity of Q_2^{-1} with respect to $|\hat{\mathbf{g}}^\dagger \boldsymbol{v}|$ as seen in P2, p is a non-increasing function of $|\hat{\mathbf{g}}^\dagger \boldsymbol{v}|$, whose minimum is 0 and maximum is $\|\hat{\mathbf{g}}\|$. As such, we have the corresponding achievable upper and lower bounds in (14a) and (14b), respectively. ∎

Remarkably, it is noted that when the transmit power p is outside the interval $[\underline{P}, \bar{P}]$, the interference constraint cannot be active. To be more specific, when $p < \underline{P}$, the interference constraint is always satisfied and can thus be ignored, while if $p > \bar{P}$, then the interference constraint can never be satisfied and such p is not permitted.

B. The Closed-Form Analytical Solution for w

Given $\|\mathbf{w}\|^2 = p$ where $\underline{P} \le p \le \bar{P}$ and that the interference constraint is active, we have

$$|\hat{\mathbf{g}}^\dagger \mathbf{w}| = Q_1^{-1}\left(\frac{\sqrt{I}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}}, 1-\varepsilon\right)\sqrt{\frac{\|\mathbf{w}\|^2 \sigma^2}{2}}. \qquad (17)$$

With this fixed p, (12) is equivalent to

$$\max_{\mathbf{w}} |\hat{\mathbf{h}}^\dagger \mathbf{w}| \quad \text{s.t.} \quad \begin{cases} \|\mathbf{w}\|^2 = p, \\ |\hat{\mathbf{g}}^\dagger \mathbf{w}| = Q_1^{-1}\left(\frac{\sqrt{I}}{\sqrt{\frac{p\sigma^2}{2}}}, 1-\varepsilon\right)\sqrt{\frac{p\sigma^2}{2}}. \end{cases}$$

$$(18)$$

The following lemma describes an important fact for the optimal solution of (18).

Lemma 2. *The optimal $\mathbf{w}$, denoted by $\mathbf{w}_{\text{opt}}$, to (18) should lie in the space spanned by $\hat{\mathbf{g}}$ and $\mathbf{g}_\perp$, i.e., $\mathbf{w}_{\text{opt}} = a\hat{\mathbf{g}} + b\mathbf{g}_\perp$, where a, b are complex scalar coefficients and*

$$\mathbf{g}_\perp \triangleq \left(\mathbf{I} - \frac{\hat{\mathbf{g}}\hat{\mathbf{g}}^\dagger}{\hat{\mathbf{g}}^\dagger \hat{\mathbf{g}}}\right)\hat{\mathbf{h}}. \qquad (19)$$

Proof: The following proof is inspired by Proposition 1 in [13].

Since $\hat{\mathbf{h}}$ is a vector of length N and $\hat{\mathbf{g}}^\dagger \mathbf{g}_\perp = 0$, the optimal solution, without loss of generality, $\mathbf{w}_{\text{opt}}$, can always be expressed as

$$\mathbf{w}_{\text{opt}} = a\hat{\mathbf{g}} + b\mathbf{g}_\perp + \mathbf{U}\boldsymbol{\nu}, \qquad (20)$$

in which $\mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_{N-2}] \in \mathbb{C}^{N\times(N-2)} \ne \mathbf{0}$ denotes the null space for $\mathbf{g}_\perp$ and $\hat{\mathbf{g}}$, so that $\mathbf{g}_\perp^\dagger \mathbf{U} = \hat{\mathbf{g}}^\dagger \mathbf{U} = \mathbf{0}$, a, b are some complex scalars, and $\boldsymbol{\nu}$ is a complex vector of length $N-2$.

Next, we like to show that the optimal solution $\mathbf{w}_{\text{opt}}$ must require $\boldsymbol{\nu} = \mathbf{0}$, and this proof is achieved by the method of contradiction. To begin, we assume that $\boldsymbol{\nu} \ne \mathbf{0}$. Then, we can always construct a new vector

$$\mathbf{w}_1 = a\hat{\mathbf{g}} + b\mathbf{g}_\perp + \delta\mathbf{g}_\perp e^{j\phi}, \qquad (21)$$

where $\phi \triangleq \arg\left(\hat{\mathbf{h}}^\dagger(a\hat{\mathbf{g}} + b\mathbf{g}_\perp)\right)$ and $\delta \ge 0$ is chosen such that $\|\mathbf{w}_1\| = \sqrt{p}$.

It is easy to check that the interference caused by $\mathbf{w}_1$ remains the same as that by $\mathbf{w}_{\text{opt}}$, i.e.,

$$\hat{\mathbf{g}}^\dagger \mathbf{w}_1 = a\hat{\mathbf{g}}^\dagger \hat{\mathbf{g}} = \hat{\mathbf{g}}^\dagger \mathbf{w}_{\text{opt}}. \qquad (22)$$

In addition, we also have

$$\begin{aligned} |\hat{\mathbf{h}}^\dagger \mathbf{w}_1| &= |\hat{\mathbf{h}}^\dagger(a\hat{\mathbf{g}} + b\mathbf{g}_\perp + \delta\mathbf{g}_\perp e^{j\phi})| \\ &= |\hat{\mathbf{h}}^\dagger(a\hat{\mathbf{g}} + b\mathbf{g}_\perp)| + \delta\hat{\mathbf{h}}^\dagger\left(\mathbf{I} - \frac{\hat{\mathbf{g}}\hat{\mathbf{g}}^\dagger}{\hat{\mathbf{g}}^\dagger \hat{\mathbf{g}}}\right)\hat{\mathbf{h}} \\ &\ge |\hat{\mathbf{h}}^\dagger(a\hat{\mathbf{g}} + b\mathbf{g}_\perp)| = |\hat{\mathbf{h}}^\dagger \mathbf{w}_{\text{opt}}|. \end{aligned} \qquad (23)$$

Therefore, if $\boldsymbol{\nu} \ne \mathbf{0}$, it is possible to further increase the objective function by employing $\mathbf{w}_1$ instead of $\mathbf{w}_{\text{opt}}$, which contradicts the optimality of $\mathbf{w}_{\text{opt}}$. The proof is completed. ∎

The problem (18) is then simplified to find the optimal scalars a and b. The simplified problem is similar to (17)–(19) in [14] and a close-form solution is straightforward using the approach there. The complete algorithm is now summarized in Algorithm 2.

V. SIMULATION RESULTS

A. Setup and Benchmark

Simulations are conducted to evaluate the performance of the proposed system in independent and identically distributed (i.i.d.) Rayleigh flat-fading channels, i.e., $\mathbf{g}_l \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \forall l$, and $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. The noise at each PU and the SU is also assumed to be zero-mean and unit-variance complex Gaussian. In addition, all channel error variances are assumed to be 0.05, i.e., $\sigma_h^2 = \sigma_l^2 = 0.05, \forall l$. The maximum transmitted signal-to-noise ratio (SNR) for the SU, defined as $\frac{P}{N_0}$, is set to be 10 (dB). The received SNR for each PU has a similar definition. We also assume that the SU transmitter has three antennas and the PU transmitter has two antennas serving two PUs, i.e., $N = 3$ and $L = 2$. Uncoded transmission and binary phase shift keying (BPSK) modulation are assumed. To produce the numerical results, for the PU network, we use zero-forcing beamforming in [15] so that no inter-user interference is present within the PU network. Results for the following benchmarks are compared: i) the non-robust method, which optimizes the system as if $\hat{\mathbf{h}}$ and $\{\hat{\mathbf{g}}_l\}$ are perfect, and ii) the worst-case based method, which is described as follows.

In the worst-case approach, the beamforming optimization at the SU transmitter is done in order that the interference

Algorithm 2 Optimal Robust Beamforming with Single Interference Constraint

1: **Input:** $\hat{\mathbf{h}}$, $\hat{\mathbf{g}}$, σ_h^2, σ^2, ΔP, and P.
2: **begin**
3:     Compute $\underline{P}$ and $\bar{P}$.
4:     **if** $P \leq \underline{P}$, **then**
5:         $\mathbf{w}_{\text{opt}}$ is given by (13).
6:         Go to line 28.
7:     **end**
8:     **if** $P \leq \bar{P}$, **then**
9:         **if** $\|\mathbf{w}\|^2 = P$ is optimal, **then**
10:             $\mathbf{w}_{\text{opt}}$ is given by (13).
11:             Go to line 28.
12:         **else**
13:             $\bar{P} := \min(\bar{P}, P)$.
14:             $p = \underline{P}$.
15:             Initialize $i = 1$.
16:             **While** $p \leq \bar{P}$
17:                 $f[i] = Q\left(\dfrac{\varsigma}{\sqrt{\frac{p\sigma_h^2}{2}}}, \dfrac{\gamma}{\sqrt{\frac{p\sigma_h^2}{2}}}\right)$.
18:                 $i := i + 1$.
19:                 $p := \underline{P} + (i-1)\Delta P$.
20:             **end**
21:             $i^* = \arg\max_i f[i]$.
22:             $p := \underline{P} + (i^*-1)\Delta P$.
23:             Find $\mathbf{w}_{\text{opt}}$ using similar method in [14].
24:         **end**
25:     **end**
26: **end**
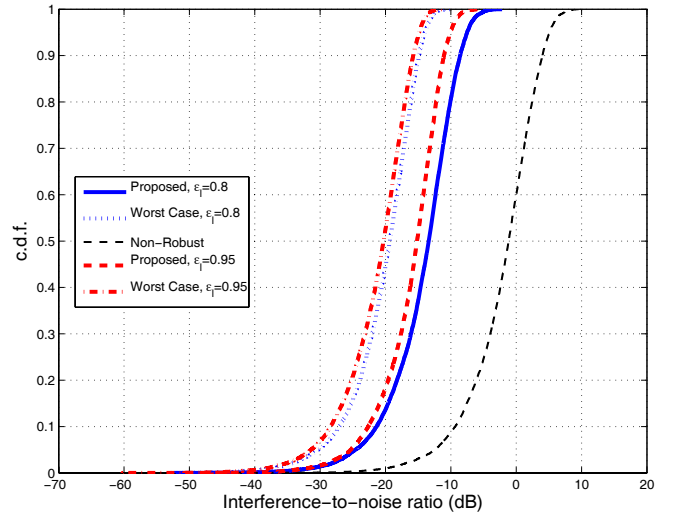27: **Output:** $\mathbf{w}_{\text{opt}}$.

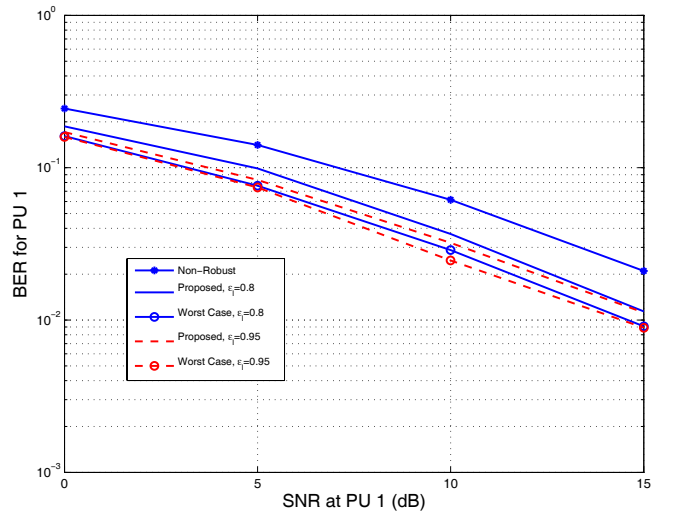Fig. 1. The c.d.f. for the interference power received at the first PU with $\frac{I_l}{N_0} = -10$ (dB) $\forall l$.

Fig. 2. The BER results for the PUs against the received SNR with $\frac{I_l}{N_0} = 0$ (dB) $\forall l$.

levels at the PUs are controlled below the required thresholds for every possible channel error realizations, i.e.,

$$\max_{\|\mathbf{w}\|^2 \leq P} \min_{\|\Delta\mathbf{h}\| \leq \xi^{(h)}} |\mathbf{h}^\dagger \mathbf{w}|^2 \quad \text{s.t.} \quad \max_{\|\Delta\mathbf{g}_l\| \leq \xi_l^{(g)}} |\mathbf{g}_l^\dagger \mathbf{w}|^2 \leq I_l \ \forall l, \tag{24}$$

for some carefully chosen $\xi^{(h)}$ and $\{\xi_l^{(g)}\}$. Note that it is possible to use an ellipsoidal region to bound the CSI errors, as in [16], and the principle is the same. Further, (24) in its current form is not convex, but can be reformulated to an SOCP problem as follows [17]:

$$\max_{\|\mathbf{w}\|^2 \leq P} \hat{\mathbf{h}}^\dagger \mathbf{w} - \xi^{(h)} \|\mathbf{w}\| \quad \text{s.t.} \quad |\hat{\mathbf{g}}_l^\dagger \mathbf{w}| \leq \sqrt{I_l} - \xi_l^{(g)} \|\mathbf{w}\| \ \forall l. \tag{25}$$

To have a fair comparison between the proposed algorithm (Algorithms 1 & 2) and the worst-case based method, we obtain the bounds $\xi^{(h)}$ and $\{\xi_l^{(g)}\}$ appropriately such that

$$\begin{cases} \text{Prob}\left(\|\Delta\mathbf{h}\| \leq \xi^{(h)}\right) = \delta \text{ for some } \delta > 0, \\ \text{Prob}\left(\|\Delta\mathbf{g}_l\| \leq \xi_l^{(g)}\right) = \varepsilon_l, \forall l. \end{cases} \tag{26}$$

It is interesting to see the similarity between the constraint in (25) and that in (9). In (9), the right-hand-side, which is given by

$$Q_1^{-1}\left(\frac{\sqrt{I_l}}{\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}}, 1 - \varepsilon_l\right)\sqrt{\frac{\|\mathbf{w}\|^2 \sigma_l^2}{2}}, \tag{27}$$

is a complicated function in I_l, σ_l^2 and $\|\mathbf{w}\|^2$, while in (25), it takes the simple form of

$$\sqrt{I_l} - \xi_l^{(g)} \|\mathbf{w}\|. \tag{28}$$

B. Results

In Fig. 1, results are provided for the cumulative distribution function (c.d.f.) of the received interference power at the first PU (or PU 1) from the SU when the interference temperature is set to -10 dB, i.e., $\frac{I_l}{N_0} = -10$ dB for $l = 1, 2$. The interference levels to the PUs are required to be 80% and 95% acceptable, or $\varepsilon_l = 0.80, 0.95 \ \forall l$. We see that the required probability that the resulting interference is below -10 dB is satisfied for the proposed scheme, while in the worst-case method, the interference power never exceeds -10 dB. Results also show that for the non-robust method, more than 90% of the time, the interference level exceeds the required -10 dB.

The effect of interference control is studied by the bit-error-rate (BER) results for PU 1 as shown in Fig. 2, where
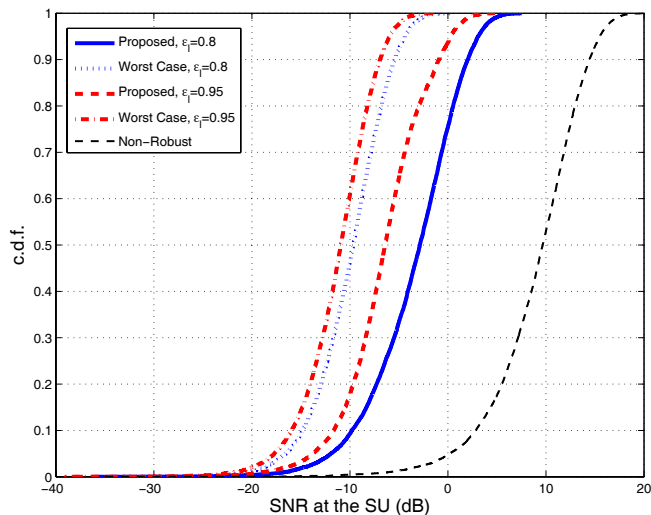
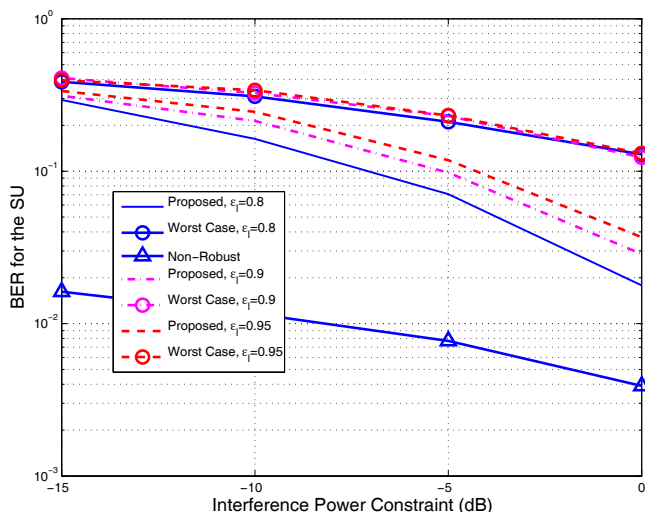Fig. 3. The c.d.f. for the received signal power at the SU with $\frac{I_l}{N_0} = -10$ (dB) $\forall l$.

Fig. 4. The BER results for the SU against the interference level.

$\frac{I_l}{N_0} = 0$ dB for $l = 1, 2$ and $\varepsilon_l = 0.80, 0.95$. As can be seen, the non-robust system results in very poor BER performance due to the errors in CSI. Considerable performance gain can be obtained using the proposed algorithm and the worst-case based approach in all received SNR regions. The worst-case approach achieves only slightly lower BER than the proposed algorithm due to the fact that the PUs in this case suffer the least interference from the SU.

Fig. 3 illustrates the c.d.f. results of the received signal power at the SU for $\frac{I_l}{N_0} = -10$ dB, $\varepsilon_l = 0.80, 0.95$, for $l = 1, 2$. Results indicate that the non-robust method outputs the strongest signal because the interference constraints are not respected, while the signal power of the worst-case approach is the weakest as it controls the interference level on every possible CSI error realizations.

The BER performance for the SU is plotted against various interference temperature requirements ranging from $\frac{I_l}{N_0} = -15 \sim 0$ dB and $\varepsilon_l = 0.80, 0.90, 0.95$, for $l = 1, 2$, in Fig. 4. It is seen that the BER for all schemes decreases

if more interference can be tolerated. Also, the worst-case approach sacrifices to gain absolute control of interference and has the worst BER performance while the non-robust method achieves the best BER performance at the cost of no control on interference to the PUs. This result aligns with that in Fig. 3. As expected, results also indicate that the performance of the SU decreases as the interference constraint becomes more strict. Combined this result with Fig. 2, we see that compared with the worst-case approach, the proposed algorithm provides much better SU performance at the cost of slight PUs performance degradation. To summarize, our results reveal that the proposed system greatly outperforms the worst-case based system and provides a means to tradeoff the performance between the PUs and the SU through service probability in an analytical and controllable way.

VI. CONCLUSION

This letter studied the stochastic robust transmit beamforming in CR to balance the interference control for PUs and signal enhancement for SU using probabilistic constraints. We showed that the problem can be optimally solved using SOCP in tandem with a simple one-dimensional search on the transmit power. For the case with only one PU and a given transmit power, a closed-form solution is possible. Simulation results illustrated that the proposed algorithm provides adjustable robustness and a controllable performance tradeoff between the PUs and the SU in CR and greatly improves SU performance over the worst-case approach with slight PUs performance degradation.

REFERENCES

[1] "Spectrum policy task force," *Federal Communications Commission*, ET Docket No. 02–135, Tech. Rep., Nov. 2002.
[2] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 6, pp. 13–18, Aug. 1999.
[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
[4] S. M. Mishra, A. Sahai, and R. W. Brodensen, "Cooperative sensing among cognitive radios," in *Proc. IEEE International Conference on Communications*, June 2006, Istanbul, Turkey, vol. 4, pp. 1658–1663.
[5] Y. C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.
[6] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.
[7] L. Zhang, Y. C. Liang, and Y. Xin, "Joint beamforming and power control for multiple access channels in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 38–51, Jan. 2008.
[8] M. H. Islam, Y. C. Liang, and A. T. Hoang, "Joint power control and beamforming for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2415–2419, July 2008.
[9] L. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, "Robust cognitive beamforming with partial channel state information," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4143–4153, Aug. 2009.
[10] H. Cox, R. M. Zeskind, and M. M. Owen, "Robust adaptive beamforming," *IEEE Trans. Signal Process.*, vol. 35, no. 10, pp. 1365–1376, Oct. 1987.
[11] K. L. Bell, Y. Ephraim, and H. L. V. Trees, "A Bayesian approach to robust adaptive beamforming," *IEEE Trans. Signal Process.*, vol. 48, no. 2, pp. 386–398, Feb. 1987.
[12] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 2000
[13] E. Jorswieck and E. Larsson, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5292–5296, Oct. 2008.

[14] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[15] Z. G. Pan, K. K. Wong, and T. S. Ng, "Generalized multiuser orthogonal space division multiplexing," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1–5, Nov. 2004.

[16] G. Zheng, K. K. Wong, and T. S. Ng, "Robust linear MIMO in the downlink: a worst-case optimization with ellipsoidal uncertainty regions," *EURASIP J. Advances Signal Process.*, vol. 2008, article ID 609028, 15 pages, 2008.

[17] S. A. Vorobyov, A. B. Gershman, and Z. Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 313–324, Feb. 2003.