Practical Physical Layer Network Coding for Two-Way Relay Channels: Performance Analysis and Comparison

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***Abstract*—This paper investigates the performance of practical physical-layer network coding (PNC) schemes for two-way relay channels. We first consider a network consisting of two source nodes and a single relay node, which is used to aid communication between the two source nodes. For this scenario, we investigate transmission over two, three or four time slots. We show that the two time slot PNC scheme offers a higher maximum sum-rate, but a lower sum-bit error rate (BER) than the four time slot transmission scheme for a number of practical scenarios. We also show that the three time slot PNC scheme offers a good compromise between the two and four time slot transmission schemes, and also achieves the best maximum sum-rate and/or sum-BER in certain practical scenarios. To facilitate comparison, we derive new closed-form expressions for the outage probability, maximum sum-rate and sum-BER. We also consider an opportunistic relaying scheme for a network with multiple relay nodes, where a single relay is chosen to maximize either the maximum sum-rate or minimize the sum-BER. Our results indicate that the opportunistic relaying scheme can significantly improve system performance, compared to a single relay network.**

***Index Terms*—Two-way relaying, physical-layer network coding.**

I. Introduction

THE use of relay transmission has been shown to offer significant performance benefits, including being able to achieve spatial diversity through node cooperation [1, 2] and extending coverage without requiring large transmitter powers. This has made them attractive options to be used in cellular, ad-hoc networks and military communications [3]. The two most common relaying protocols are Decode and Forward (DF) and Amplify and Forward (AF). The AF protocol is a simple scheme, which amplifys the signal transmitted from the source and forwards it to the destination [4–6], and unlike the DF protocol, no decoding at the relay is performed.

In this paper, we consider a two-way wireless system where two source nodes, A and B, communicate with each other through the aid of relay node(s) using an AF protocol. We first consider the two-way relay network with a single relay node, and then extend our results to a network with multiple relays. In a two-way relay transmission scheme, communication can take place over four time slots, where source A communicates to source B in the first two time slots with source B remaining idle, and source B communicates to source A in the last two time slots with source A remaining idle. We refer to this scheme as a four time slot transmission scheme, and its performance has been studied in Rayleigh fading environments when taking into account various transmit signal-to-noise ratio (SNR) variations at the source and relay [7, 8]. This has been extended to Nakagami-m fading environments in [9], when the transmit SNRs at the source and relay are the same, and in [10] when the two transmit SNRs are different. One problem with this transmission scheme is the relatively low maximum sum-rate, as a consequence of transmission over four time slots [11, 12].

The two source nodes can also communicate over two time slots, where in the first time slot, source A and B simultaneously transmit to the relay, while in the second time slot, the relay amplifies and forwards the received signals to both sources. Such a scheme is referred to as a two time slot physical-layer network coding (PNC) scheme and has been shown to achieve higher maximum sum-rates than the four time slot transmission scheme, due to less time slots being used for transmission [11, 12]. In contrast to the four time slot transmission scheme, there are limited analytical results for the two time slot PNC scheme. In [13, 14], the two time slot PNC scheme was proposed using an AF protocol for single antennas at the relay, and in [15] where multiple antennas were used at the relay for linear processing. However, in the above papers, no analytical results were derived for the maximum sum-rate or sum-bit error rate (BER), ie. the sum of the BER at source A and B. In [16], the two time slot PNC scheme was considered using the Denoise-aNd-Forward (DNF) protocol, however, the AF protocol was not considered and analytical results were limited to the BER using BPSK modulation only. In [11], analytical expressions were derived for the maximum sum-rate using the AF protocol, but was shown to be accurate only for small relay, or large total transmit powers. In addition, the outage probability was not considered, and analytical BER expressions were not derived. Further, comparison of the sum-BER between the two time slot PNC and four time slot transmission scheme was not considered.

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Finally, the two source nodes can communicate over three time slots, which we refer to as a three time slot PNC scheme. In this transmission scheme, source A and B transmit to the relay over the first and second time slot, respectively. In the third time slot, the relay transmits a function of the received signals to source A and B. As with the two time slot PNC scheme, there are limited analytical results for the three time slot PNC scheme using the AF protocol. In [17, 18], the three time slot PNC scheme was proposed using the DF protocol, however the AF protocol was not considered. In [14], a three time slot PNC scheme was considered using the DF protocol, while in [13], the AF protocol was considered. However, the above papers utilizing the AF protocol considered a three time slot PNC scheme where the relay transmits a sum of the received signals to the source nodes. In contrast, we propose a scheme where each received signal from source A and B is weighted by a power allocation number at the relay, such that a particular performance measure is optimized. We consider using the power allocation numbers to either maximize the maximum sum-rate or minimize the sum-BER.

In this paper, we conduct new performance analysis on the two and three time slot PNC, and four time slot transmission schemes, which allow us to obtain important insights into these schemes. In particular, we first derive exact closed-form expressions for the outage probability. These will be used to derive new closed-form expressions for the sum-BER, and a tight upper bound for the maximum sum-rate. By applying these results, we show that the two time slot PNC scheme has a higher maximum sum-rate than the four time slot scheme for a number of practical scenarios, including when the source nodes transmit at high SNR. Furthermore, we also show that the sum-BER of the four time slot is lower than the two time slot PNC scheme for a number of practical scenarios. This suggests a positive tradeoff between the two and four time slot transmission scheme, in either optimizing the maximum sum-rate or the sum-BER.

We then show that the performance of the three time slot PNC scheme has a performance which lies between the two and four time slot transmission scheme for various scenarios, and as such offers a good compromise between the two schemes. In addition, for a number of practical scenarios, we show that the three time slot PNC scheme can offer a better performance than both the two and four time slot transmission schemes. We show that this holds in particular when the transmit powers from source A and B are different, as the three time slot PNC scheme will allocate more power at the relay to the received signal from the source with the larger transmit power. This is in contrast to the two and four time slot scheme, which weights the received signals from the two sources equally, and thus cannot exploit the asymmetries in the source powers to improve performance.

Finally, we extend the two, three and four time slot schemes to incorporate multiple relays. Specifically, we consider an opportunistic relay scheme, where one relay is chosen such that either the maximum sum-rate is maximized, or the sum-BER is minimized. We derive new sum-BER and maximum sum-rate scaling laws expressions for the two and four time slot transmission schemes. We also compare the three transmission schemes, and show that a substantial performance improvement can be obtained by the use of the opportunistic relay scheme.

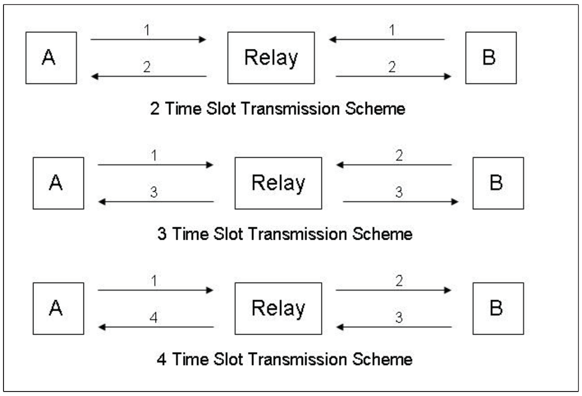

Fig. 1. Block diagram of the two, three and four time slot schemes.

The rest of the paper is organized as follows. We first describe the two, three and four time slot schemes in Section II. We then derive new closed-form expressions for the outage probability, maximum sum-rate and sum-BER in Section III. Using these expressions, we compare the three transmission schemes in Section IV. Next, we analyze the opportunistic relaying scheme for the two, three and four time slot schemes in Section V. We finish off with a conclusion in Section VI.

II. System Model

Consider a wireless two-way network where two source nodes, A and B, communicate with each other through a single relay node using an AF protocol. In particular, source A and B transmit, respectively, data symbols x_A and x_B with power P_A and P_B. In addition, the Rayleigh channel between source A and the relay, and source B and the relay are denoted by $h \sim \mathcal{CN}(0,1)$ and $g \sim \mathcal{CN}(0,1)$ respectively, while the additive white Gaussian noise (AWGN) at source A, source B and the relay are denoted by $n_A \sim \mathcal{CN}(0,\sigma_A^2)$, $n_B \sim \mathcal{CN}(0,\sigma_B^2)$ and $n_r \sim \mathcal{CN}(0,\sigma_r^2)$ respectively.

The received signal(s) at the relay from source A and B are multiplied by a gain G. The gain is chosen with the aid of the instantaneous channel state information (CSI), h and g, and the noise statistics, such that the instantaneous transmit power at the relay is constrained. We will refer to this gain as the channel-noise-assisted AF (CNA-AF) gain. At high SNR, the noise becomes negligible, and the gain only relies on the instantaneous CSI. We will refer to this gain as the channel-assisted (CA-AF) gain. The signal is then further multiplied by power P_r, and then forwarded to the source nodes. For convenience, we denote $\bar{\gamma}_A = \frac{P_A}{\sigma_r^2}$, $\bar{\gamma}_B = \frac{P_B}{\sigma_r^2}$, $\bar{\gamma}_{r,A} = \frac{P_r}{\sigma_A^2}$ and $\bar{\gamma}_{r,B} = \frac{P_r}{\sigma_B^2}$.

We consider three transmission schemes, which differ in the number of time slots used for the source nodes to communicate with each other through the relay node, as shown in Fig. 1. In particular, we consider total transmission over either two, three or four time slots. We analyze the performance of these three transmissions schemes in the next two sections, demonstrating that each may outperform the other, depending on different system parameters.

We assume that the channel and noise are constant during these time slots, and that the channels associated with each source are known at that source, ie. h is known at source A and g is known at source B. Further, we also assume that $G\sqrt{P_r}$ is known at both source nodes. We start by describing the two, three and four time slot transmission schemes.

A. Two Time Slot PNC Scheme

In the first transmission scheme, source A and B communicate with each other over two time slots. In the first time slot, the two source nodes simultaneously transmit to the relay node. In the second time slot, the relay node amplifies and forwards the received signals to the two source nodes. The received signal in the first time slot at the relay node can be written as

$$r = \sqrt{P_A} h x_A + \sqrt{P_B} g x_B + n_r \ . \tag{1}$$

The received signals in the second time slot at source A and B are given, respectively, by

$$\begin{aligned} y_A = G\sqrt{P_r} h r + n_A &= G\sqrt{P_r}\sqrt{P_B} h g x_B \\ &+ G\sqrt{P_r}\sqrt{P_A} h h x_A + G\sqrt{P_r} h n_r + n_A \end{aligned} \tag{2}$$

and

$$\begin{aligned} y_B = G\sqrt{P_r} g r + n_B &= G\sqrt{P_r}\sqrt{P_A} g h x_A \\ &+ G\sqrt{P_r}\sqrt{P_B} g g x_B + G\sqrt{P_r} g n_r + n_B \ . \end{aligned} \tag{3}$$

Since each source node knows its transmitted signal, it then cancels the self interference term, from which the resulting signals at source A and B can be written as

$$y_A^* = G\sqrt{P_r}\sqrt{P_B} h g x_B + G\sqrt{P_r} h n_r + n_A \tag{4}$$

and

$$y_B^* = G\sqrt{P_r}\sqrt{P_A} g h x_A + G\sqrt{P_r} g n_r + n_B \ . \tag{5}$$

We can thus write the output SNRs at source A and B respectively as

$$\begin{aligned} \gamma_{A,\mathrm{2TS}} &= \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B |g|^2 |h|^2}{\bar{\gamma}_{r,A}|h|^2 + \frac{1}{G^2\sigma_r^2}} \quad \text{and} \\ \gamma_{B,\mathrm{2TS}} &= \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A |h|^2 |g|^2}{\bar{\gamma}_{r,B}|g|^2 + \frac{1}{G^2\sigma_r^2}} \ . \end{aligned} \tag{6}$$

The gain is given by

$$G = \frac{1}{\sqrt{P_A|h|^2 + P_B|g|^2 + \zeta\sigma_r^2}} \tag{7}$$

where $\zeta = 1$ corresponds to the CNA-AF gain, and $\zeta = 0$ corresponds to the CA-AF gain. Note that the two gains converge at high $\bar{\gamma}_A$ or $\bar{\gamma}_B$. The resultant output SNRs at source A and B are obtained by substituting (7) into (6), and are given respectively by

$$\gamma_{A,\mathrm{2TS}} = \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B |g|^2 |h|^2}{(\bar{\gamma}_{r,A} + \bar{\gamma}_A)|h|^2 + \bar{\gamma}_B |g|^2 + \zeta} \tag{8}$$

and

$$\gamma_{B,\mathrm{2TS}} = \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A |h|^2 |g|^2}{(\bar{\gamma}_{r,B} + \bar{\gamma}_B)|g|^2 + \bar{\gamma}_A |h|^2 + \zeta} \ . \tag{9}$$

B. Three Time Slot PNC Scheme

In the second transmission scheme, we consider the use of three time slots for communication between source A and B. In the first time slot, source A transmits to the relay node. In the second time slot, source B transmits to the relay node. In the third time slot, the relay node processes the received signals, and amplifies and forwards the processed signal to the two source nodes. The received signals in the first and second time slots at the relay can be written, respectively, as

$$r_A = \sqrt{P_A} h x_A + n_{r1} \ \text{ and } \ r_B = \sqrt{P_B} g x_B + n_{r2} \tag{10}$$

where n_{r1} and n_{r2} are independent and identically distributed AWGN, with distribution given by $\sim \mathcal{CN}(0, \sigma_r^2)$. The received signals in the third time slot at source A and B are given respectively by

$$\begin{aligned} y_A &= \sqrt{P_r} G h \left(\alpha_B r_A + \alpha_A r_B\right) + n_A \\ &= G\sqrt{P_r}\sqrt{P_A}\alpha_B h h x_A + G\sqrt{P_r}\alpha_B h n_{r1} \\ &\quad + G\sqrt{P_r}\sqrt{P_B}\alpha_A h g x_B + G\sqrt{P_r}\alpha_A h n_{r2} + n_A \end{aligned} \tag{11}$$

and

$$\begin{aligned} y_B &= \sqrt{P_r} G g \left(\alpha_B r_A + \alpha_A r_B\right) + n_B \\ &= G\sqrt{P_r}\sqrt{P_A}\alpha_B g h x_A + G\sqrt{P_r}\alpha_B g n_{r1} \\ &\quad + G\sqrt{P_r}\sqrt{P_B}\alpha_A g g x_B + G\sqrt{P_r}\alpha_A g n_{r2} + n_B \end{aligned} \tag{12}$$

where α_A and α_B are power allocation numbers such that $\alpha_A^2 + \alpha_B^2 = 1$. The power allocation numbers are chosen to optimize certain performance measures described in the next section. As we will show in the following sections, the power allocation number can make a significant positive impact on the overall system performance. This is particularly the case when the transmit powers from source A and B are different, as in this scenario, the overall system performance is increased if the received signal with the larger transmit power is weighted more, as opposed to equal power allocation in the two and four time slot schemes.

Each source node then cancels the self interference term, where we assume each source node has knowledge of α_A and α_B. We note that the power allocation at the relay, and obtaining the knowledge of α_A and α_B at the source nodes results in a slight complexity increase in the system. The resulting signals at source A and B are given, respectively, by

$$\begin{aligned} y_A^* &= G\sqrt{P_r}\sqrt{P_B}\alpha_A h g x_B \\ &\quad + G\sqrt{P_r} h(\alpha_B n_{r1} + \alpha_A n_{r2}) + n_A \end{aligned} \tag{13}$$

and

$$\begin{aligned} y_B^* &= G\sqrt{P_r}\sqrt{P_A}\alpha_B g h x_A \\ &\quad + G\sqrt{P_r} g(\alpha_B n_{r1} + \alpha_A n_{r2}) + n_B \ . \end{aligned} \tag{14}$$

The output SNRs at source A and B can be written, respectively, as

$$\begin{aligned} \gamma_{A,\mathrm{3TS}} &= \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B \alpha_A^2 |h|^2 |g|^2}{\bar{\gamma}_{r,A}|h|^2 + \frac{1}{G^2\sigma_r^2}} \quad \text{and} \\ \gamma_{B,\mathrm{3TS}} &= \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A \alpha_B^2 |g|^2 |h|^2}{\bar{\gamma}_{r,B}|g|^2 + \frac{1}{G^2\sigma_r^2}} \ . \end{aligned} \tag{15}$$

The gain is given by

$$G^2 = \frac{1}{\alpha_B^2 P_A |h|^2 + \alpha_A^2 P_B |g|^2 + \sigma_r^2 \zeta} \quad (16)$$

where $\zeta = 1$ corresponds to the CNA-AF gain, and $\zeta = 0$ corresponds to the CA-AF gain. Note that the two gains converge at high $\bar{\gamma}_A$ or $\bar{\gamma}_B$. The output SNRs at source A and B are given by substituting (16) into (15), and are given respectively by

$$\gamma_{A,3\text{TS}} = \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B \alpha_A^2 |h|^2 |g|^2}{(\bar{\gamma}_{r,A} + \alpha_B^2 \bar{\gamma}_A)|h|^2 + \alpha_A^2 \bar{\gamma}_B |g|^2 + \zeta} \quad (17)$$

and

$$\gamma_{B,3\text{TS}} = \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A \alpha_B^2 |g|^2 |h|^2}{(\bar{\gamma}_{r,B} + \alpha_A^2 \bar{\gamma}_B)|g|^2 + \alpha_B^2 \bar{\gamma}_A |h|^2 + \zeta} \ . \quad (18)$$

C. Four Time Slot Transmission

In the third transmission scheme, source A and B communicate with each other over four time slots. In the first time slot, source A transmits to the relay node. In the second time slot, the relay node amplifies and forwards the received signal to source B. In the third time slot, source B transmits to the relay node. In the fourth time slot, the relay node amplifies and forwards the received signal to source A. For fair comparison, we use the same total source and relay powers in the two, three and four time slot transmission schemes. Thus for the four time slot transmission scheme, the relays transmit with power $\frac{P_r}{2}$ during the second and fourth time slot. The received signals in the first and third time slots at the relay are given in (10). The received signals in the second and fourth time slots at source A and B are given, respectively, by

$$y_A = \sqrt{\frac{P_r}{2}}\sqrt{P_B} G_B h g x_B + \sqrt{\frac{P_r}{2}} G_B h n_{r1} + n_A \quad (19)$$

$$y_B = \sqrt{\frac{P_r}{2}}\sqrt{P_A} G_A g h x_A + \sqrt{\frac{P_r}{2}} G_A g n_{r2} + n_B \ . \quad (20)$$

The output SNRs at source A and B can be written, respectively, as

$$\gamma_{A,4\text{TS}} = \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B |h|^2 |g|^2}{\bar{\gamma}_{r,A}|h|^2 + \frac{2}{G_B^2 \sigma_r^2}} \quad \text{and}$$

$$\gamma_{B,4\text{TS}} = \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A |h|^2 |g|^2}{\bar{\gamma}_{r,B}|g|^2 + \frac{2}{G_A^2 \sigma_r^2}} \ . \quad (21)$$

The gains are given by

$$G_A^2 = \frac{1}{P_A |h|^2 + \zeta \sigma_r^2} \quad \text{and} \quad G_B^2 = \frac{1}{P_B |g|^2 + \zeta \sigma_r^2} \quad (22)$$

where $\zeta = 1$ corresponds to the CNA-AF gain, and $\zeta = 0$ corresponds to the CA-AF gain. Note that the two gains converge at high $\bar{\gamma}_A$ or $\bar{\gamma}_B$. The output SNRs at source A and B are obtained by substituting (22) into (21) and are given respectively by

$$\gamma_{A,4\text{TS}} = \frac{\bar{\gamma}_{r,A}\bar{\gamma}_B |h|^2 |g|^2}{\bar{\gamma}_{r,A}|h|^2 + 2\bar{\gamma}_B |g|^2 + 2\zeta} \quad (23)$$

and

$$\gamma_{B,4\text{TS}} = \frac{\bar{\gamma}_{r,B}\bar{\gamma}_A |h|^2 |g|^2}{\bar{\gamma}_{r,B}|g|^2 + 2\bar{\gamma}_A |h|^2 + 2\zeta} \ . \quad (24)$$

Note that for the two, three and four time slot transmission schemes, we can incorporate different relay positions by appropriately scaling the average transmit SNR at the sources and relay. Specifically, if d_A is the distance between source A and the relay, and d_B the distance between source B and the relay, then the average transmit SNRs, which takes into account the relay positions, can be written as

$$\bar{\gamma}_{r,A}^* = \frac{\bar{\gamma}_{r,A}}{d_A^\delta}, \bar{\gamma}_{r,B}^* = \frac{\bar{\gamma}_{r,B}}{d_B^\delta}, \bar{\gamma}_A^* = \frac{\bar{\gamma}_A}{d_A^\delta}, \bar{\gamma}_B^* = \frac{\bar{\gamma}_B}{d_B^\delta} \quad (25)$$

where $\tau = 2, 3$ or 4 and δ is the path loss exponent.

In the next section, we derive new outage probability, sum-BER and maximum sum-rate expressions for the three transmission schemes. These expressions will be used to analyze the relative performance and obtain key insights for the three transmission schemes.

III. Performance Analysis

A. Received SNR

To facilitate the analysis of the different schemes, we first make some preliminary observations on the received SNR at source A and source B. For the two and three time slot PNC schemes, we see in (8), (9), (17) and (18) that the received SNRs at source A and B are dependent on both $\bar{\gamma}_A$ and $\bar{\gamma}_B$. Further, whilst increasing $\bar{\gamma}_A$ will *increase* the received SNR at source A, it will also *decrease* the received SNR at source B. The same effect can be observed when increasing $\bar{\gamma}_B$. It is thus not clear whether the overall system performance, ie. the combined performance at source A and B, is improved for varying $\bar{\gamma}_A$ when $\bar{\gamma}_B$ is held constant, or varying $\bar{\gamma}_B$ when $\bar{\gamma}_A$ is held constant. This will be explored analytically in this section, and through numerical analysis in the following section. In contrast to the PNC schemes, we see in (23) and (24) that the four time slot transmission scheme results in a received SNR at source A and B that is independent, respectively, of $\bar{\gamma}_A$ and $\bar{\gamma}_B$. This is directly due to transmission of the data from source A and B at different time slots. Thus increasing $\bar{\gamma}_A$ or $\bar{\gamma}_B$ will always result in a better overall system performance.

We now consider the case when $\bar{\gamma} = \bar{\gamma}_A = \bar{\gamma}_B$. For the two and four time slot transmission scheme, we see in (8), (9), (23) and (24) that this will result in source A and B having the same average performance. Further, at high source and relay powers (when $\zeta = 0$), note that the relative performance of the two and four time slot schemes is dependent on the transmit power at the relay. When $\bar{\gamma}_{r,A} = \bar{\gamma}_{r,B} = \bar{\gamma}$, we see in (8), (9), (23) and (24) that the two and four time slot schemes have the same average received SNR. However, this is not the case when $\bar{\gamma}_{r,A} \neq \bar{\gamma}$ and $\bar{\gamma}_{r,B} \neq \bar{\gamma}$. In this scenario, it is not clear whether the two or four time slot scheme performs better. This will be discussed in this section. In contrast to the two and four time slot schemes, we see in (17) and (18) that the the three time slot PNC scheme results in a received SNR that is not the same at source A and B. Rather the performance is then dependent on the power allocation numbers α_A and α_B. We will also explore the impact of this later in this section.

B. Outage Probability

We first derive new outage probability expressions for the three transmission schemes. These will be used to derive sum-BER and maximum sum-rate expressions later in this section. The outage probability for the received SNR is an important quality of service measure defined as the probability that the received SNR drops below an acceptable SNR threshold $\gamma_{\rm th}$. We denote the outage probability of the received SNR at source ϱ for the τ time slot transmission scheme as $F_{\gamma_{\varrho,\tau\rm TS}}(\gamma_{\rm th})$.

1) Exact: We first present a new exact closed-form outage probability expression for the three transmission schemes. Note that the outage probability for the four time slot transmission scheme has been derived in [9]. However we present the results here for completeness.

Theorem 1: The outage probability at source A and B for the two, three and four time slot transmission schemes are given by substituting (26), (27) and (28) at the top of the next page into

$$F_{\gamma_{\varrho,\tau\rm TS}}(\gamma_{\rm th}) = 1 - \frac{2\sqrt{\gamma_{\rm th}\left(\gamma_{\rm th}\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS} + \beta_{\varrho,\tau\rm TS}\zeta\right)}}{\beta_{\varrho,\tau\rm TS}} e^{-\frac{\gamma_{\rm th}(\phi_{\varrho,\tau\rm TS}+\theta_{\varrho,\tau\rm TS})}{\beta_{\varrho,\tau\rm TS}}} K_1\left(\frac{2\sqrt{\gamma_{\rm th}\left(\gamma_{\rm th}\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS} + \beta_{\varrho,\tau\rm TS}\zeta\right)}}{\beta_{\varrho,\tau\rm TS}}\right) \tag{29}$$

where $\bar{\gamma}_{A,3\rm TS} = \frac{\bar{\gamma}_A}{\alpha_A^2+\alpha_B^2}$, $\bar{\gamma}_{B,3\rm TS} = \frac{\bar{\gamma}_B}{\alpha_A^2+\alpha_B^2}$ and $K_1(\cdot)$ is the modified Bessel function of the second kind [19, Eq. (9.6.2)]. Further, $\varrho = A$ or B, $\tau = 2, 3$ or 4 and $\zeta = 1$ for the CNA-AF gain and $\zeta = 0$ for the CA-AF gain.

Proof: See Appendix A. ■

Note that *Theorem 1* presents the outage probability for the *received SNR*. It is also of interest to consider the maximum rate. If the instantaneous maximum rates at source A and B for the τ time slot scheme are given respectively by

$$R_{A,\tau\rm TS} = \frac{1}{\tau}\log_2(1+\gamma_{A,\tau\rm TS}) \quad \text{and} \quad R_{B,\tau\rm TS} = \frac{1}{\tau}\log_2(1+\gamma_{B,\tau\rm TS}), \tag{30}$$

then the outage probability for the *maximum rate* for the τ time slot transmission scheme at source ϱ, defined as the probability that the maximum rate drops below a threshold $R_{\rm th}$, can be shown from (30) to be given by

$$F_{R_{\varrho,\tau\rm TS}}(R) = F_{\gamma_{\varrho,\tau\rm TS}}\left(2^{R_{\rm th}\tau} - 1\right). \tag{31}$$

2) Lower Bound: We now present a new outage probability lower bound expression. This will be useful in deriving accurate maximum sum-rate expressions later in this section.

Lemma 1: The outage probability at source A and B for the two, three and four time slot transmission schemes, for both the CNA-AF and CA-AF gains, can be lower bounded by substituting (26), (27) and (28) into

$$F_{\gamma_{\varrho,\tau\rm TS}}(\gamma_{\rm th}) \geq F_{\gamma^{\rm lb}_{\varrho,\tau\rm TS}}(\gamma_{\rm th}) = 1 - e^{-\frac{\gamma_{\rm th}\left(\theta_{\varrho,\tau\rm TS}+\phi_{\varrho,\tau\rm TS}\right)}{\beta_{\varrho,\tau\rm TS}}} \tag{32}$$

where $\varrho = A$ or B and $\tau = 2, 3$ or 4.

Proof: See Appendix B. ■

3) High SNR, Low Outage: We now present new asymptotic outage probability expressions at high SNR, or low outage probabilities, for the two, three and four time slot transmission schemes. This will be useful in analyzing the sum-BER at high SNR later in this section.

Lemma 2: The asymptotic outage probability at source A and B for the two, three and four time slot transmission schemes, for both the CNA-AF and CA-AF gains, are obtained by substituting (26), (27) and (28) into

$$F_{\gamma_{\varrho,\tau\rm TS}}(\gamma_{\rm th}) = \frac{(\theta_{\varrho,\tau\rm TS}+\phi_{\varrho,\tau\rm TS})\gamma_{\rm th}}{\beta_{\varrho,\tau\rm TS}} + o\left(\gamma_{\rm th}^2\right) \tag{33}$$

where $\varrho = A$ or B and $\tau = 2, 3$ or 4.

Proof: We first note that the outage probability can be written in the general form

$$\Pr\left(\frac{\beta_{\varrho,\tau\rm TS}}{\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS}}\frac{\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS}|h|^2|g|^2}{\theta_{\varrho,\tau\rm TS}|g|^2+\phi_{\varrho,\tau\rm TS}|h|^2} \leq \gamma_{\rm th}\right) = \Pr\left(\frac{\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS}|h|^2|g|^2}{\theta_{\varrho,\tau\rm TS}|g|^2+\phi_{\varrho,\tau\rm TS}|h|^2} \leq \frac{\gamma_{\rm th}(\theta_{\varrho,\tau\rm TS}\phi_{\varrho,\tau\rm TS})}{\beta_{\varrho,\tau\rm TS}}\right). \tag{34}$$

We proceed by using the asymptotic outage probability result derived in [20], followed by some algebraic manipulation. Note that this also corresponds to the asymptotic outage probability, which can be derived by using the outage probability lower bound in (32). ■

C. Sum-Bit Error Rate

We now derive new expressions for the sum-BER, which is defined as the sum of the BER at source A and B. We denote the sum-BER for the τ time slot transmission scheme as $P_{b,\tau\rm TS}$. In order to derive expressions for the sum-BER, let us first consider the sum-symbol error rate (SER), defined as the sum of the SER at source A and B, and given by

$$P_{s,\tau\rm TS} = \mathbf{E}\left[a\mathcal{Q}\left(\sqrt{2b\gamma_{A,\tau\rm TS}}\right)\right] + \mathbf{E}\left[a\mathcal{Q}\left(\sqrt{2b\gamma_{B,\tau\rm TS}}\right)\right] \tag{35}$$

where $\mathcal{Q}(\cdot)$ is the Gaussian-$\mathcal{Q}$ function defined as $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}{\rm d}y$ and a and b are modulation specific constants. Such modulation formats include BPSK ($a = 1, b = 1$), BFSK with orthogonal signalling ($a = 1, b = 0.5$) or minimum correlation ($a = 1, b = 0.715$) and M-ary PAM $\left(a = 2(M-1)/M, b = 3/(M^2-1)\right)$ [21]. Our new results also provide the sum-BER to other modulation formats for which (35) can be shown to be a tight upper bound, e.g. M-PSK ($a = 2, b = \sin^2(\pi/M)$) and M-QAM $\left(a = 4\left(1 - 1/\sqrt{M}\right), b = 3/2(M-1)\right)$ [21]. Note that the upper bound in (35) for M-PSK and M-QAM is asymptotically tight at high SNR. To derive expressions for the sum-BER as a function of the SNR per bit when using M-ary signalling, we use the common approximation [21, 22]

$$P_{b,\tau\rm TS} \approx \frac{P_{s,\tau\rm TS}}{\log_2(M)}, \quad \bar{\gamma}^{\rm b}_\varrho = \frac{\bar{\gamma}_\varrho}{\log_2(M)}, \quad \bar{\gamma}^{\rm b}_{r,\varrho} = \frac{\bar{\gamma}_{r,\varrho}}{\log_2(M)} \quad \text{and} \quad \bar{\gamma}^{\rm b}_{\varrho,\tau\rm TS} = \frac{\bar{\gamma}_{\varrho,\tau\rm TS}}{\log_2(M)} \tag{36}$$

$$\begin{aligned} &\beta_{A,2\mathrm{TS}} = \bar{\gamma}_{r,A}\bar{\gamma}_B, \;\; \theta_{A,2\mathrm{TS}} = \bar{\gamma}_{r,A} + \bar{\gamma}_A, \;\; \phi_{A,2\mathrm{TS}} = \bar{\gamma}_B \qquad \text{Source } A \\ &\beta_{B,2\mathrm{TS}} = \bar{\gamma}_{r,B}\bar{\gamma}_A, \;\; \theta_{B,2\mathrm{TS}} = \bar{\gamma}_{r,B} + \bar{\gamma}_B, \;\; \phi_{B,2\mathrm{TS}} = \bar{\gamma}_A \qquad \text{Source } B\,, \end{aligned} \tag{26}$$

$$\begin{aligned} &\beta_{A,3\mathrm{TS}} = \bar{\gamma}_{r,A}\bar{\gamma}_{B,3\mathrm{TS}}\alpha_A^2, \;\; \theta_{A,3\mathrm{TS}} = \bar{\gamma}_{r,A} + \alpha_B^2\bar{\gamma}_{A,3\mathrm{TS}}, \;\; \phi_{A,3\mathrm{TS}} = \alpha_A^2\bar{\gamma}_{B,3\mathrm{TS}} \qquad \text{Source } A \\ &\beta_{B,3\mathrm{TS}} = \bar{\gamma}_{r,B}\bar{\gamma}_{A,3\mathrm{TS}}\alpha_B^2, \;\; \theta_{B,3\mathrm{TS}} = \bar{\gamma}_{r,B} + \alpha_A^2\bar{\gamma}_{B,3\mathrm{TS}}, \;\; \phi_{B,3\mathrm{TS}} = \alpha_B^2\bar{\gamma}_{A,3\mathrm{TS}} \qquad \text{Source } B \end{aligned} \tag{27}$$

$$\begin{aligned} &\beta_{A,4\mathrm{TS}} = \bar{\gamma}_{r,A}\bar{\gamma}_B, \;\; \theta_{A,4\mathrm{TS}} = \bar{\gamma}_{r,A}, \;\; \phi_{A,4\mathrm{TS}} = 2\bar{\gamma}_B \qquad \text{Source } A \\ &\beta_{B,4\mathrm{TS}} = \bar{\gamma}_{r,B}\bar{\gamma}_A, \;\; \theta_{B,4\mathrm{TS}} = \bar{\gamma}_{r,B}, \;\; \phi_{B,4\mathrm{TS}} = 2\bar{\gamma}_A \qquad \text{Source } B \end{aligned} \tag{28}$$

where $\varrho = A$ or B, $\tau = 2, 3$ or 4 and $\bar{\gamma}_{r,\varrho}^{\mathrm{b}}$ and $\bar{\gamma}_{\varrho,\tau\mathrm{TS}}^{\mathrm{b}}$ denotes the average transmit SNR per bit at the relay and two sources respectively.

For the three time slot PNC scheme, we consider two choices for the power allocation numbers when considering the sum-BER. The first choice involves choosing the allocation numbers such that the average sum-BER in (36) is minimized, ie. the sum-BER averaged over the fading channels h and g. In this paper, we use the term sum-BER and average sum-BER interchangeably, unless explicitly stated. In this respect, we can use the expressions we derive in this subsection to numerically calculate the optimal power allocation numbers. The second choice involves choosing the allocation numbers such that the instantaneous sum-BER is minimized, ie. the sum-BER taking into account the instantaneous channel h and g. Although it is difficult to derive closed form expressions for the instantaneous sum-BER, we numerically investigate its performance using Monte Carlo simulations in the next section.

1) Arbitrary SNR: We first present new approximate sum-BER expressions for the three transmission schemes at arbitrary SNR values.

Lemma 3: The sum-BER at source A and B for the two, three and four time slot transmission schemes, for both the CNA-AF and CA-AF gains, can be approximated by substituting (26), (27) and (28) into

$$P_{b,\tau\mathrm{TS}} \approx \frac{a}{\log_2(M)} - \frac{3a\sqrt{b}\sqrt{\pi}}{16\log_2(M)} \left(\frac{(\kappa_{2,A}-\kappa_{1,A})^2}{\kappa_{2,A}^{\frac{5}{2}}} {}_2F_1\left(\frac{5}{2},\frac{3}{2};2;\frac{\kappa_{1,A}}{\kappa_{2,A}}\right) - \frac{(\kappa_{2,B}-\kappa_{1,B})^2}{\kappa_{2,B}^{\frac{5}{2}}} {}_2F_1\left(\frac{5}{2},\frac{3}{2};2;\frac{\kappa_{1,B}}{\kappa_{2,B}}\right)\right) \tag{37}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is is the Gauss hypergeometric function [19, Eq. (9.6.2)],

$$\begin{aligned} \kappa_{1,A} &= \frac{\theta_{A,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{A,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{A,\tau\mathrm{TS}}^{\mathrm{b}}} + b - \frac{2\sqrt{\theta_{A,\tau\mathrm{TS}}^{\mathrm{b}}\phi_{A,\tau\mathrm{TS}}^{\mathrm{b}}}}{\beta_{A,\tau\mathrm{TS}}^{\mathrm{b}}}, \\ \kappa_{2,A} &= \frac{\theta_{A,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{A,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{A,\tau\mathrm{TS}}^{\mathrm{b}}} + b + \frac{2\sqrt{\theta_{A,\tau\mathrm{TS}}^{\mathrm{b}}\phi_{A,\tau\mathrm{TS}}^{\mathrm{b}}}}{\beta_{A,\tau\mathrm{TS}}^{\mathrm{b}}}, \end{aligned} \tag{38}$$

$$\begin{aligned} \kappa_{1,B} &= \frac{\theta_{B,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{B,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{B,\tau\mathrm{TS}}^{\mathrm{b}}} + b - \frac{2\sqrt{\theta_{B,\tau\mathrm{TS}}^{\mathrm{b}}\phi_{B,\tau\mathrm{TS}}^{\mathrm{b}}}}{\beta_{B,\tau\mathrm{TS}}^{\mathrm{b}}}, \\ \kappa_{2,B} &= \frac{\theta_{B,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{B,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{B,\tau\mathrm{TS}}^{\mathrm{b}}} + b + \frac{2\sqrt{\theta_{B,\tau\mathrm{TS}}^{\mathrm{b}}\phi_{B,\tau\mathrm{TS}}^{\mathrm{b}}}}{\beta_{B,\tau\mathrm{TS}}^{\mathrm{b}}} \end{aligned} \tag{39}$$

and $\tau = 2, 3$ or 4. Further, $\theta_{\varrho,\tau\mathrm{TS}}^{\mathrm{b}}$, $\beta_{\varrho,\tau\mathrm{TS}}^{\mathrm{b}}$ and $\phi_{\varrho,\tau\mathrm{TS}}^{\mathrm{b}}$ are obtained by substituting $\bar{\gamma}_\varrho = \bar{\gamma}_\varrho^{\mathrm{b}}\log_2(M)$, $\bar{\gamma}_{r,\varrho} = \bar{\gamma}_{r,\varrho}^{\mathrm{b}}\log_2(M)$, and $\bar{\gamma}_{\varrho,\tau\mathrm{TS}} = \bar{\gamma}_{\varrho,\tau\mathrm{TS}}^{\mathrm{b}}\log_2(M)$ into $\theta_{\varrho,\tau\mathrm{TS}}$, $\beta_{\varrho,\tau\mathrm{TS}}$ and $\phi_{\varrho,\tau\mathrm{TS}}$ respectively in (26), (27) and (28) where $\varrho = A$ or B.

Proof: We begin by rewriting the sum-BER expression given in (35) directly in terms of outage probability at source A and B, using integration by parts, as follows

$$P_{s,\tau\mathrm{TS}} = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-bu}}{\sqrt{u}} \left(F_{\gamma_{A,\tau\mathrm{TS}}}(u) + F_{\gamma_{B,\tau\mathrm{TS}}}(u)\right) \mathrm{d}u \tag{40}$$

where $\tau = 2, 3, 4$. We then proceed by substituting (29) with $\zeta = 0$ into (40), and solving the resultant integral using [19, Eq. (6.621.3)]. Finally, by applying the approximations in (36), we obtain the desired result. ■

Note that the proof of the sum-BER involved the use of the CA-AF gain only, which we consider for mathematical tractability. However, we note that the CA-AF gain provides a tight upper bound for the CNA-AF gain, with the bound tightening for increasing SNR.

2) High SNR: To gain insights, we consider the sum-BER at high SNR. Note that in the high SNR regime, the performance using the CNA-AF gain approaches the performance using the CA-AF gain.

Lemma 4: The sum-BER at high SNR for the two, three and four time slot transmission schemes, for both the CNA-AF and CA-AF gains, can be approximated by substituting (26), (27) and (28) into

$$P_{b,\tau\mathrm{TS}}^{\infty} = \frac{a}{4b\log_2(M)} \left(\frac{\theta_{A,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{A,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{A,\tau\mathrm{TS}}^{\mathrm{b}}} + \frac{\theta_{B,\tau\mathrm{TS}}^{\mathrm{b}} + \phi_{B,\tau\mathrm{TS}}^{\mathrm{b}}}{\beta_{B,\tau\mathrm{TS}}^{\mathrm{b}}} \right) \tag{41}$$

where $\tau = 2, 3$ or 4.

Proof: The result follows by using a general SISO BER result from [23] combined with Lemma 2. ■

Note it can be shown that, for a fixed $\bar{\gamma}_r = \bar{\gamma}_{r,A} = \bar{\gamma}_{r,B}$, the diversity order of the three transmission schemes is one.

Corollary 1: For equal noise variances, ie. $\sigma_A^2 = \sigma_B^2$, and at high SNR, the difference between the sum-BER of the two time slot PNC and the four time slot transmission scheme is given by

$$P_{b,2\mathrm{TS}}^{\infty} - P_{b,4\mathrm{TS}}^{\infty} = \frac{1}{4}\left(\frac{a_2}{b_2 \log_2^2(M_2)} - \frac{a_4}{b_4 \log_2^2(M_4)}\right) \times \left(\frac{1}{\bar{\gamma}_B^b} + \frac{1}{\bar{\gamma}_A^b}\right) + \frac{1}{4\bar{\gamma}_r}\left(\frac{a_2}{b_2 \log_2^2(M_2)}\left(\frac{\bar{\gamma}_A^b}{\bar{\gamma}_B^b} + \frac{\bar{\gamma}_B^b}{\bar{\gamma}_A^b} + 2\right) - \frac{4a_4}{b_4 \log_2^2(M_4)}\right) \quad (42)$$

where $\bar{\gamma}_r = \bar{\gamma}_{r,A} = \bar{\gamma}_{r,B}$.

Proof: The proof follows by substituting (26) and (28), with $\sigma_A^2 = \sigma_B^2$, into (41) for the two and four time slot transmission scheme respectively, and evaluating the difference. ■

Corollary 1 allows us to obtain key insights into the relative performance of the two and four time slot transmission schemes in various practical scenarios. We see that when source A and B use the same power, i.e. $\bar{\gamma}^b = \bar{\gamma}_A^b = \bar{\gamma}_B^b$, the expression in (42) reduces to

$$P_{b,2\mathrm{TS}}^{\infty} - P_{b,4\mathrm{TS}}^{\infty} = \left(\frac{1}{\bar{\gamma}_r} + \frac{1}{2\bar{\gamma}^b}\right)\left(\frac{a_2}{b_2 \log_2^2(M_2)} - \frac{a_4}{b_4 \log_2^2(M_4)}\right). \quad (43)$$

We see in (43) that when the source powers are the same, the dominating factor in the relative performance of the two schemes is the modulation format. When the modulation is the same for the two and four time slot schemes, the sum-BER for both schemes at high SNRs are identical. However, this will not be a proper comparison as the spectral efficiency of the two schemes are different. This will be explored in the following example:

Example: To maintain the same spectral efficiency, we consider the use of 4-QAM modulation for the two time slot PNC scheme and 16-QAM for the four time slot transmission scheme. This corresponds to $a_2 = 2, b_2 = \frac{1}{2}, M_2 = 4$ and $a_4 = 3$, $b_4 = \frac{1}{10}, M_4 = 16$ for 4-QAM and 16-QAM respectively. Substituting these parameters into (42), we have

$$P_{b,2\mathrm{TS}}^{\infty} - P_{b,4\mathrm{TS}}^{\infty} = -\frac{7}{32}\left(\frac{1}{\bar{\gamma}_B^b} + \frac{1}{\bar{\gamma}_A^b}\right) + \frac{1}{4\bar{\gamma}_r}\left(\frac{\bar{\gamma}_A^b}{\bar{\gamma}_B^b} + \frac{\bar{\gamma}_B^b}{\bar{\gamma}_A^b} - \frac{13}{2}\right). \quad (44)$$

We see in (44) that a sufficient condition that the four time slot scheme performs better than the two time slot scheme is when the transmit SNRs at the source are far apart, ie. $\bar{\gamma}_A^b >> \bar{\gamma}_B^b$ or $\bar{\gamma}_B^b >> \bar{\gamma}_A^b$. This can occur, for example, when the source A to relay distance is greater than the source B to relay distance, or vice versa. One explanation for this is because, as discussed in Section III-A, the received SNRs at both source A and B for the two time slot PNC scheme are sensitive to both $\bar{\gamma}_A^b$ and $\bar{\gamma}_B^b$. Clearly, we see that the disparity between $\bar{\gamma}_A^b$ and $\bar{\gamma}_B^b$ results in an overall negative performance for the two time slot scheme compared to the four time slot scheme. When $\bar{\gamma}_A^b$ is very close to $\bar{\gamma}_B^b$, we see that the two time slot PNC scheme performs better than the four time slot transmission scheme. As shown in (43), this is due to the different modulation formats used for the two schemes, resulting in $\frac{a_2}{b_2 \log_2^2(M_2)} \leq \frac{a_4}{b_4 \log_2^2(M_4)}$.

Corollary 2: For the three time slot PNC scheme, the power allocation numbers which minimizes the sum-BER at high SNR are given by

$$\alpha_A^2 = \frac{\sqrt{\frac{\bar{\gamma}_B(\bar{\gamma}_{r,B}+\bar{\gamma}_B)}{\bar{\gamma}_{r,B}}}}{\sqrt{\frac{\bar{\gamma}_B(\bar{\gamma}_{r,B}+\bar{\gamma}_B)}{\bar{\gamma}_{r,B}}} + \sqrt{\frac{\bar{\gamma}_A(\bar{\gamma}_{r,A}+\bar{\gamma}_A)}{\bar{\gamma}_{r,A}}}} \quad \text{and}$$

$$\alpha_B^2 = \frac{\sqrt{\frac{\bar{\gamma}_A(\bar{\gamma}_{r,A}+\bar{\gamma}_A)}{\bar{\gamma}_{r,A}}}}{\sqrt{\frac{\bar{\gamma}_B(\bar{\gamma}_{r,B}+\bar{\gamma}_B)}{\bar{\gamma}_{r,B}}} + \sqrt{\frac{\bar{\gamma}_A(\bar{\gamma}_{r,A}+\bar{\gamma}_A)}{\bar{\gamma}_{r,A}}}}. \quad (45)$$

Proof: This is given by substituting (27) into (41), and taking the derivative w.r.t. α_B. ■

We see in (45) that if $\bar{\gamma}_A = \bar{\gamma}_B$ and $\bar{\gamma}_{r,A} = \bar{\gamma}_{r,B}$, then equal power allocation is optimal. We also see that α_B and α_A monotonically increases with P_A and P_B respectively. Thus for a fixed total source power, ie. $\bar{\gamma}_A + \bar{\gamma}_B$, more relay power will be allocated to the signal received from the source with the largest transmit power. Note that the optimal power allocation given in (45) is not the optimal power allocation which minimizes the instantaneous sum-BER. However, as we will see in the next section, the insights we gain from (45) still hold when we minimize the instantaneous sum-BER.

D. Maximum Sum-Rate

We now present results for the maximum sum-rate, which is defined as the sum of the maximum rate at source A and B. We denote the maximum sum-rate for the τ time slot transmission scheme as $R_{\tau\mathrm{TS}}$, and is given by

$$R_{\tau\mathrm{TS}} = \frac{1}{\tau}\left(\mathbf{E}_{\gamma_{A,\tau\mathrm{TS}}}\left[\log_2(1+\gamma)\right] + \mathbf{E}_{\gamma_{B,\tau\mathrm{TS}}}\left[\log_2(1+\gamma)\right]\right) \quad (46)$$

where $\tau = 2, 3$ or 4.

For the three time slot PNC scheme, we consider two choices for the power allocation numbers when considering the maximum sum-rate. The first choice involves choosing the allocation numbers such that the average maximum sum-rate in (46) is maximized, ie. the maximum sum-rate averaged over the fading channels h and g. In this paper, we use the term maximum sum-rate and average maximum sum-rate interchangeably, unless explicitly stated. In this respect, we can use the expressions we derive in this subsection to numerically calculate the optimal power allocation numbers. The second choice involves choosing the allocation numbers such that the instantaneous maximum sum-rate is maximized, ie. the maximum sum-rate taking into account the instantaneous channel h and g. Although it is difficult to derive closed form maximum sum-rate expressions for this case, we numerically investigate its performance using Monte Carlo simulations in the next section.

Unfortunately, finding exact expressions for the maximum sum-rate in (46) using the exact distribution in (29) is difficult, hence we focus on deriving an upper bound, presented in the following lemma, which we show to be tight in the next section.

Lemma 5: The maximum sum-rate for the two, three and four time slot transmission schemes, for both the CNA-AF and CA-AF gains, can be upper bounded by substituting (26), (27) and (28) into

$$R^{\mathrm{ub}}_{\tau\mathrm{TS}} = \frac{\log_2(e)}{\tau}\left(e^{\frac{\theta_{A,\tau\mathrm{TS}}+\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}} E_1\left(\frac{\theta_{A,\tau\mathrm{TS}}+\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}\right) + e^{\frac{\theta_{B,\tau\mathrm{TS}}+\phi_{B,\tau\mathrm{TS}}}{\beta_{B,\tau\mathrm{TS}}}} E_1\left(\frac{\theta_{B,\tau\mathrm{TS}}+\phi_{B,\tau\mathrm{TS}}}{\beta_{B,\tau\mathrm{TS}}}\right)\right) \tag{47}$$

where $E_1(\cdot)$ is the exponential integral [19, Eq. (5.1.1)].

Proof: The proof follows by using the outage probability upper bound in (32) with (46), and applying the integral identity in [24]. ∎

Corollary 3: At high SNR, the difference between the maximum sum-rate of the two and the four time slot transmission scheme is given by

$$R_{\mathrm{single,2TS}} - R_{\mathrm{single,4TS}} = \log_2(e)\left(2\ln(\bar{\gamma})\right) + \log_2(e)\left(\ln\left(\frac{(\omega_r|h|^2+\omega_B|g|^2)\omega_r\omega_B|g|^2|h|^2}{((\omega_r+\omega_A)|h|^2+\omega_B|g|^2)^2} \times \frac{(\omega_r|g|^2+\omega_A|h|^2)\omega_r\omega_A|h|^2|g|^2}{((\omega_r+\omega_B)|g|^2+\omega_A|h|^2)^2}\right)\right) \tag{48}$$

where $\bar{\gamma} = \bar{\gamma}_A + \bar{\gamma}_B + \bar{\gamma}_r$, $\bar{\gamma}_A = \omega_A\bar{\gamma}$, $\bar{\gamma}_B = \omega_B\bar{\gamma}$, $\bar{\gamma}_r = \omega_r\bar{\gamma}$ and the expectation is taken with respect to (w.r.t.) h and g.

Proof: The proof follows by substituting (8), (9), (23) and (24) into (46), and taking the difference between the maximum sum-rate of the two and four time slot transmission schemes. ∎

It can be shown from (48) that for sufficiently large total source and relay powers $\bar{\gamma}$, the two time slot PNC scheme performs better than the four time slot transmission scheme. In contrast to the sum-BER results, the two time slot scheme always performs better than the four time slot scheme, irrespective of the particular source power combination. This is due to the fact that the number of time slots used for transmission is the dominating factor in the maximum sum-rate at high SNR.

In this section, we presented new expressions for the sum-BER and maximum sum-rate. By using these analytical expressions, we showed that the two and four time slot transmission schemes may outperform each other, depending on different system parameters. Clearly, depending on whether we are considering the maximum sum-rate or the sum-BER, and on the particular transmit powers at the sources and relay, the choice of the optimal transmission scheme is different. In the next section, we further investigate the relative performance of these two transmission schemes, and also show that the three time slot PNC scheme offers a good compromise between the two and four time slot transmission schemes, and may outperform the two in some scenarios.

IV. Numerical Analysis for Single Relay Networks

In this section, we compare the sum-BER and maximum sum-rate of the two, three and time slot transmission schemes. We investigate two scenarios based on various source powers and relay positions. For the sum-BER curves, to maintain the same spectral efficiency, we use QPSK, 8-PSK and 16-QAM for the two, three and four time slot transmission schemes respectively. Further, the sum-BER curves are plotted for the two and four time slot schemes using the analytical curves only, and not Monte Carlo simulations. This is to allow the different schemes to be clearly shown without confusion. Note that for the three time slot PNC scheme, the allocation numbers used to maximize the maximum sum-rate may not necessarily correspond to the same allocation numbers used to minimize the sum-BER, and vice-versa. In this section, we will thus also analyze the performance of the maximum sum-rate using the allocation numbers used to minimize the sum-BER, and the performance of the sum-BER using the allocation numbers used to maximize the maximum sum-rate.

A. Variable Source Powers

We first compare the maximum sum-rate and sum-BER performance of the three transmission schemes for varying source power combinations. To enable comparison of the three transmission schemes for variable source powers, we fix the total power at the two sources, denoted by $\bar{\gamma} = \bar{\gamma}_A + \bar{\gamma}_B$.

Fig. 2 shows a plot of the sum-BER vs. $\bar{\gamma}_A$ of the three transmission schemes for varying source power combinations. The analytical curves for the three transmission schemes are based on (37). For the three time slot PNC scheme, we choose the power allocation numbers such that the instantaneous sum-BER is minimized for the '3TS - Monte Carlo, Instantaneous' curves, the maximum sum-rate is maximized for the '3TS - Monte Carlo, Max. Sum-Rate' curves and the average sum-BER in (37) is minimized for the '3TS - Analytical' curves. We see that at moderate to high SNR, the three time slot PNC schemes performs better than the two and four time slot transmission schemes when $\bar{\gamma}_B = 1.5\bar{\gamma}_A$. In addition, the two time slot PNC scheme performs the worst at moderate to high SNR, as predicted by our analysis in Section III. We see that for the case $\bar{\gamma}_B = 0.5\bar{\gamma}_A$, the '3TS - Monte Carlo, Instantaneous' performs the best at high SNR, followed by the four time slot transmission scheme. Again, we see that the two time slot PNC scheme performs the worst at moderate to high SNR. We note that we have discussed the case when $\bar{\gamma}_A = \bar{\gamma}_B$ in the previous section, and do not present numerical results for this case as it does not lead to any new insights.

Fig. 3 shows a plot of the optimal power allocation numbers, used in Fig. 2, vs. $\bar{\sigma}_A$, using the three time slot PNC scheme for varying source power combinations. We see that the three curves diverge as the SNR goes larger. However, we note that they all follow the same trend, ie. either increasing or decreasing with SNR, for the particular source power combination. As predicted by our analytical analysis in Section III, we see that at high SNR, the source with the larger transmit power will be weighted more at the relay than the source with the smaller transmit power. This highlights the fact that the three time slot PNC scheme is particularly effective when the transmit powers at source A and B are different. This is in contrast to the two and four time slot schemes, which weighs the received signals from source A and B equally, and thus cannot exploit the asymmetries in the source powers to improve performance.

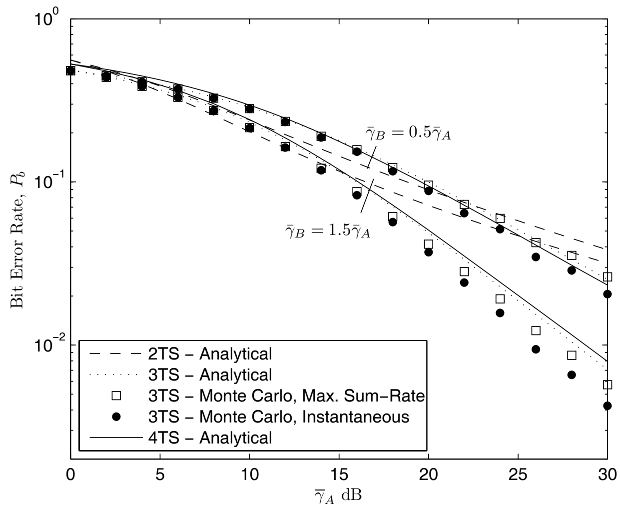

Fig. 2. Sum-BER of the two, three and four time slot schemes with $\bar{\gamma}_r = 0.8\bar{\gamma}_A$ and $\sigma_A^2 = \sigma_B^2 = 1$.

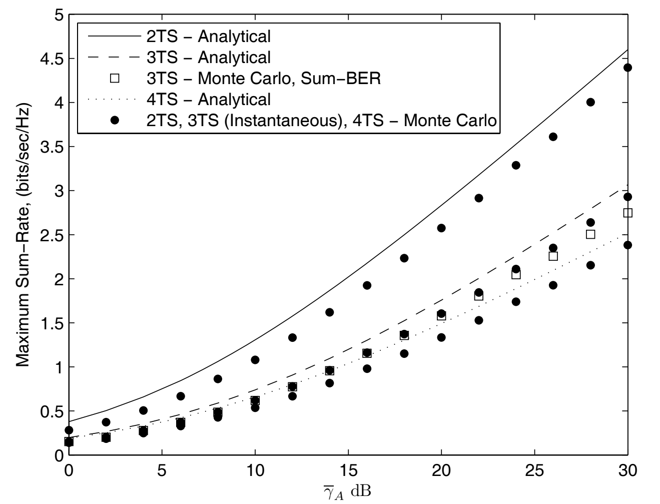

Fig. 4. Maximum sum-rate of the two, three and four time slot schemes with $\bar{\gamma}_B = 0.5\bar{\gamma}_A$ and $\bar{\gamma}_r = 0.8\bar{\gamma}_A$.

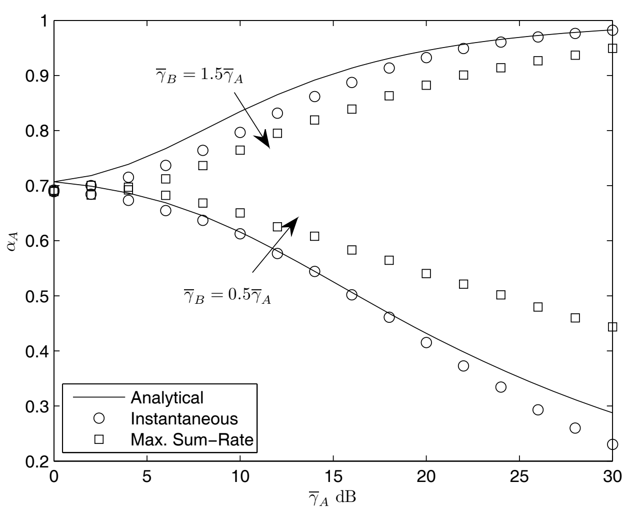

Fig. 3. Power allocation number of the three time slot PNC scheme with $\bar{\gamma}_r = 0.8\bar{\gamma}_A$ and $\sigma_A^2 = \sigma_B^2 = 1$.

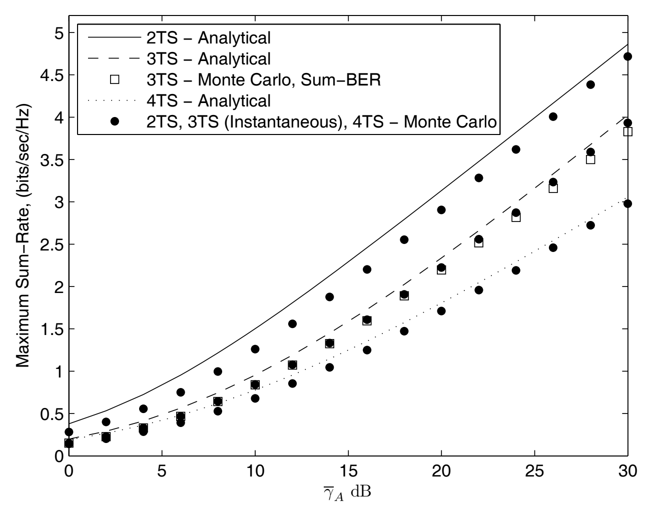

Fig. 5. Maximum sum-rate of the two, three and four time slot schemes with $\bar{\gamma}_B = 1.5\bar{\gamma}_A$ and $\bar{\gamma}_r = 0.8\bar{\gamma}_A$.

Figs. 4 and 5 show a plot of the maximum sum-rate vs. $\bar{\gamma}_A$ of the three transmission schemes for varying source power combinations. The analytical upper bound curves are from (47). Note that although for the two time slot scheme, the bound is not as tight as the other schemes, we observe that our analytical curves follow the same trends as the Monte Carlo simulated curves. This is also the case for all maximum sum-rate curves in this paper. For the three time slot PNC scheme, we choose the power allocation numbers such that the instantaneous maximum sum-rate is maximized for the Monte Carlo curves, the sum-BER is minimized for the '3TS - Monte Carlo, Sum-BER' curves and the average maximum sum-rate in (47) is maximized for the analytical curves. We see that for the three time slot PNC schemes, the instantaneous maximum sum-rate always performs better than the '3TS - Monte Carlo, Sum-BER' and '3TS - Monte Carlo, Sum-BER' sum-rate curves, as expected. We see that the maximum sum-rate of the two time slot PNC scheme performs better than the four time slot transmission scheme not only for high SNR, as predicted by our analytical results in Section III, but also for low SNR. We also see that the maximum sum-rate of the three time slot PNC scheme performs between the two and four time slot transmissions schemes. Finally, we observe that the curves in Fig. 4 are always higher than the corresponding curves in Fig. 5, due to a higher transmission power at source B in Fig. 5 than at source B in Fig. 4.

B. Variable Relay Positions

We now compare the three transmission schemes for variable relay positions. As explained in Section II, our model can incorporate relay positions by appropriate scaling of the average transmit SNRs, as given in (25). To enable comparison of the three transmission schemes for different relay positions, we fix the distance between source A and B, denoted by d,

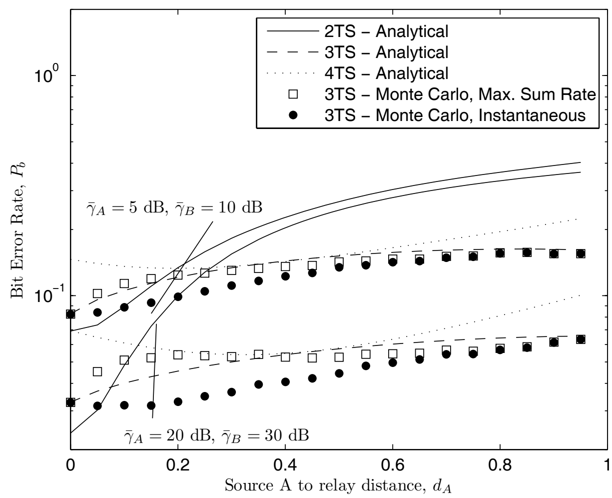

Fig. 6. Sum-BER of the two, three and four time slot schemes with $d = 1$, $\bar{\gamma}_r = \bar{\gamma}_A$ dB, $\delta = 3$ and $\sigma_A^2 = \sigma_B^2 = 1$.

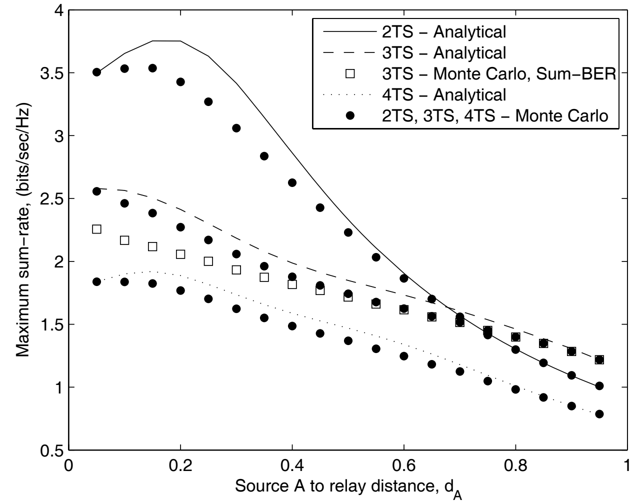

Fig. 7. Maximum sum-rate of the two, three and four time slot schemes with $\bar{\gamma}_r = 5$, $\bar{\gamma}_A = 5$, $\bar{\gamma}_B = 10$, $d = 1$, $\delta = 3$ and $\sigma_A^2 = \sigma_B^2 = 1$. The solid lines show the Analytical curves and the circles show Monte Carlo simulated curves.

such that $d = d_A + d_B$, where d_A is the distance between source A and the relay, and d_B is the distance between source B and the relay.

Fig. 6 shows a plot of the sum-BER of the three transmission schemes for different values of d_A, with $\bar{\gamma}_r = \bar{\gamma}_A$. The analytical curves for the three transmission schemes are from (37). For the three time slot PNC scheme, we choose the power allocation numbers such that the instantaneous sum-BER is minimized for the '3TS - Monte Carlo, Instantaneous' curves, the maximum sum-rate is maximized for the '3TS - Monte Carlo, Max. Sum-Rate' curves and the sum-BER in (37) is minimized for the '3TS - Analytical' curves. For the case when $\bar{\gamma}_A = 20$ dB and $\bar{\gamma}_B = 30$ dB, we see that the two time PNC transmission scheme performs best for low source A to relay distances d_A, but worst for most other distances. For other distances, we see that the '3TS - Monte Carlo, Instantaneous' scheme has the best performance. For the case when $\bar{\gamma}_A = 5$ dB and $\bar{\gamma}_B = 10$ dB, we observe again that the two time slot PNC scheme performs best only for low source A to relay distances. We also see that the crossover point of the two and four time slot schemes is higher in this scenario, than in the case when $\bar{\gamma}_A = 20$ dB and $\bar{\gamma}_B = 30$ dB. As discussed in Section III-C2, this is due to the fact that for large ratios $\frac{\bar{\gamma}_B}{\bar{\gamma}_A}$. the four time slot scheme performs better, while for small ratios, the two time slot scheme performs better. Again, we see that the three time slot PNC scheme performs better than the four time slot transmission scheme for most distances.

Figs. 7 and 8 show a plot of the maximum sum-rate of the three transmission schemes for different values of d_A, with $\bar{\gamma}_A = 5$, $\bar{\gamma}_B = 10$ and $\bar{\gamma}_A = 20$, $\bar{\gamma}_B = 30$ respectively. The analytical upper bound curves are from (47), which we see closely match the Monte Carlo simulated curves. For the three time slot PNC scheme, we choose the power allocation numbers such that the instantaneous maximum sum-rate is maximized for the Monte Carlo curves, the sum-BER is minimized for the '3TS - Monte Carlo, Sum-BER' curves and the maximum sum-rate in (47) is maximized for the analytical curves. For the two, three and four time slot transmission schemes, we see that the optimal relay position occurs when $d_A \leq 0.2$. We also see that the two time slot PNC scheme performs best for all distances, except at high d_A. Further, we observe a d_A crossover point between the two and three time slot PNC schemes, such that the two time slot performs better than the three time slot PNC scheme below the d_A crossover point, and better above it. We see that the three time slot PNC scheme performs the best in Fig. 7 when $0.65 \leq d_A \leq 1$, and in Fig. 8 when $0.6 \leq d_A \leq 1$. Since the source powers in Fig. 8 are larger than the source powers in Fig. 7, we see that the value of the d_A crossover point decreases for increasing source powers, and thus increasing the source powers results in the three time slot PNC scheme performing the best for more relay positions. This can be explained by first noting that in Figs. 7 and 8, the power at source B is greater than the power at source A. As such, when $0.5 \leq d_A \leq 1$, the power received at the relay from source B can be significantly greater than the power received from source A. In such a scenario, the impact of power allocation can be significant, and as demonstrated in Figs. 7 and 8, results in the three time slot PNC scheme performing the best, particularly in Fig. 8 where there is relatively more power received from source B than source A.

We have shown in this section that the two time slot PNC scheme performs better than the four time slot transmission scheme in terms of maximum sum-rate for all system parameters used. Furthermore, for most practical scenarios, including moderate to high SNRs, and/or moderate to large source A to relay distances, the four time slot transmission scheme performs better than the two time slot PNC scheme in terms of sum-BER. Moreover, we see that for certain source power and relay positions, the three time slot PNC scheme can perform better than both the two and four time slot transmission scheme. In addition, the three time slot PNC scheme has a sum-BER and maximum sum-rate which lies between, or

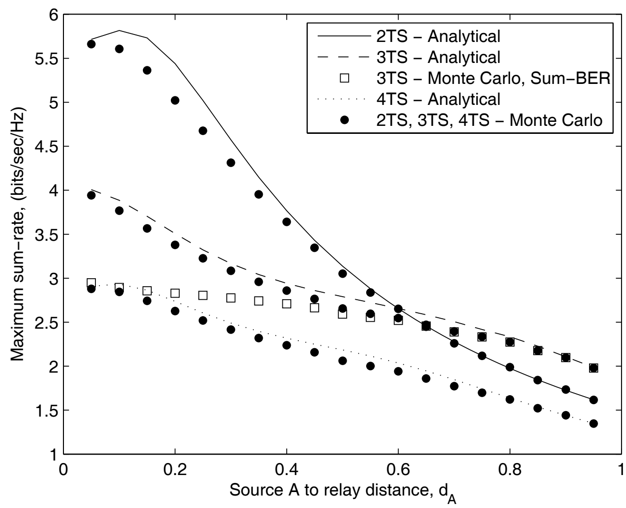

Fig. 8. Maximum sum-rate of the two, three and four time slot schemes with $\bar{\gamma}_r = 20$, $\bar{\gamma}_A = 20$, $\bar{\gamma}_B = 30$, $d = 1$, $\delta = 3$ and $\sigma_A^2 = \sigma_B^2 = 1$. The solid lines show the Analytical curves and the circles show Monte Carlo simulated curves.

performs better than, the two and four time slot transmission scheme, and as such offers a good compromise for the two and four time slot transmission schemes.

V. Opportunistic Relay Selection for Multi-Relay Two-Way Networks

We now consider a system where there are K relay nodes. We consider opportunistically selecting one out of the K relays, which is used to aid communication between the two source nodes. In particular, the relay node is chosen such that either the sum-BER is minimized or the maximum sum-rate is maximized. In general, finding an exact expression for the maximum sum-rate and sum-BER is difficult. Hence we focus on deriving results for specific cases. Furthermore, analytical performance measures of the three time slot PNC scheme does not seem tractable. Hence we investigate its performance by Monte Carlo simulations in the next section.

A. Sum-Bit Error Rate

In this subsection, the relay node is chosen such that the sum-BER is minimized. We define the sum-BER for the τ time slot transmission scheme as $P_{b,\tau\mathrm{TS}}^{\mathrm{multi}}$.

Lemma 6: At high SNR with K relays, the sum-BER for the two, three and four time slot transmission schemes are defined by substituting (26), (27) and (28) respectively into

$$P_{b,\tau\mathrm{TS}}^{\mathrm{multi},\infty} = \frac{a\Gamma\left(K+\frac{1}{2}\right)}{2\sqrt{\pi}b^K}\left(\left(\frac{\theta_{A,\tau\mathrm{TS}}+\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}\right)^K + \left(\frac{\theta_{B,\tau\mathrm{TS}}+\phi_{B,\tau\mathrm{TS}}}{\beta_{B,\tau\mathrm{TS}}}\right)^K\right). \quad (49)$$

Proof: The proof follows by deriving a similar proof to Lemma 4, and by using standard results from order statistics. ■

Note that for the three transmission schemes, for a fixed $\bar{\gamma}_r$, it can be shown in (49) that the diversity order is K.

Corollary 4: For equal noise variances, ie. $\sigma_A^2 = \sigma_B^2$, and at high SNR, the difference between the sum-BER of the two time slot PNC and the four time slot opportunistic transmission scheme is given by

$$\begin{aligned} P_{b,2\mathrm{TS}}^{\infty} - P_{S,4\mathrm{TS}}^{\infty} = {} & \frac{a_2}{4b_2\log_2^{K+1}(M_2)} \\ & + \left(\left(\frac{\bar{\gamma}_r^b+\bar{\gamma}_A^b+\bar{\gamma}_B^b}{\bar{\gamma}_r^b\bar{\gamma}_B^b}\right)^K \left(\frac{\bar{\gamma}_r^b+\bar{\gamma}_B^b+\bar{\gamma}_A^b}{\bar{\gamma}_r\bar{\gamma}_A^b}\right)^K\right) \\ & - \frac{a_4}{4b_4\log_2^{K+1}(M_4)}\left(\left(\frac{\bar{\gamma}_r^b+2\bar{\gamma}_B^b}{\bar{\gamma}_r^b\bar{\gamma}_B^b}\right)^K + \left(\frac{\bar{\gamma}_r^b+2\bar{\gamma}_A^b}{\bar{\gamma}_r^b\bar{\gamma}_A^b}\right)^K\right) \end{aligned} \quad (50)$$

where $\bar{\gamma}_r^b = \bar{\gamma}_{r,A}^b = \bar{\gamma}_{r,B}^b$.

Corollary 4 allows us to obtain key insights into the relative performance of the two and four time slot transmission schemes in various practical scenarios. To explore this further, we present the following example.

Example: To maintain the same spectral efficiency, we consider the use of 4-QAM modulation for the two time slot PNC scheme and 16-QAM for the four time slot transmission scheme. This corresponds to $a_2 = 2, b_2 = \frac{1}{2}, M_2 = 4$ and $a_4 = 3$, $b_4 = \frac{1}{10}, M_4 = 16$ for 4-QAM and 16-QAM respectively. Furthermore, we consider the case when $\bar{\gamma}^b = \bar{\gamma}_A^b = \bar{\gamma}_B^b = 2\bar{\gamma}_r^b$, which is representative of practical scenarios where the relay has less power than the two source nodes. Substituting these parameters into (50), we have

$$P_{S,2\mathrm{TS}}^{\infty} - P_{S,4\mathrm{TS}}^{\infty} = \left(\frac{5}{2\bar{\gamma}^b}\right)^K\left(1-\frac{15}{2^{K+2}}\right). \quad (51)$$

We see in (51) that a sufficient condition that $P_{S,2\mathrm{TS}}^{\infty} - P_{S,4\mathrm{TS}}^{\infty}$ is positive is when $K > 2$. Thus the four time slot transmission scheme always performs better than the two time slot PNC scheme at high SNR for the modulation schemes employed when $K > 2$.

B. Maximum Sum-Rate

In this subsection, we consider an opportunistic relay selection scheme where the relay node is chosen such that the maximum sum-rate is maximized. We denote the maximum sum-rate for the τ time slot transmission scheme as $R_{\tau\mathrm{TS}}^{\mathrm{multi}}$.

Obtaining general closed form expressions for the maximum sum-rate is difficult, due to the dependence of the output SNRs at source A and B. We hence focus on deriving the maximum sum-rate for a large number of relays, when $P_A = P_B$ and $\sigma_A^2 = \sigma_B^2$, given by the following lemma

Lemma 7: For sufficiently large K and when $P_A = P_B$ and $\sigma_A^2 = \sigma_B^2$, the maximum sum-rate is obtained by substituting (26) and (28) into

$$R_{\tau\mathrm{TS}}^{\mathrm{multi}} = \frac{1}{\tau}\left(\log_2\left(\frac{\beta_{\tau\mathrm{TS}}}{(\sqrt{\phi_{\tau\mathrm{TS}}}+\sqrt{\theta_{\tau\mathrm{TS}}})^2}\right) + \log_2\ln(K)\right) + \mathcal{O}\left(\log_2\ln\ln(K)\right) \quad (52)$$

where $\beta_{\tau\mathrm{TS}} = \beta_{A,\tau\mathrm{TS}} = \beta_{B,\tau\mathrm{TS}}$, $\phi_{\tau\mathrm{TS}} = \phi_{A,\tau\mathrm{TS}} = \phi_{B,\tau\mathrm{TS}}$ and $\theta_{\tau\mathrm{TS}} = \theta_{A,\tau\mathrm{TS}} = \theta_{B,\tau\mathrm{TS}}$.

Proof: See Appendix C. ■

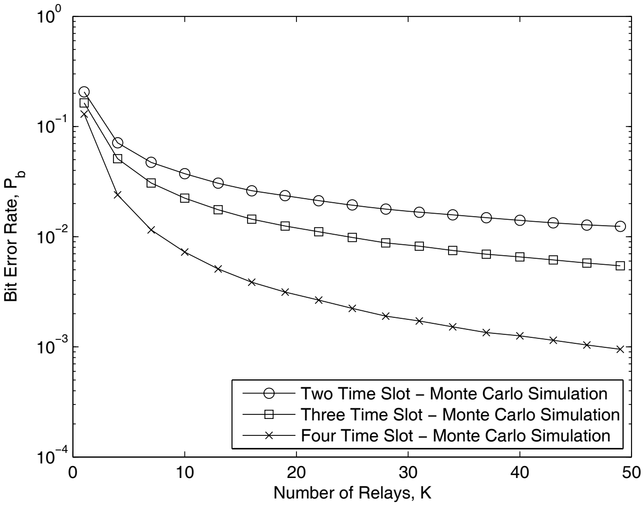

Fig. 9. Sum-BER of the two, three and four time slot opportunistic relay scheme with $\bar{\gamma}_A = \bar{\gamma}_r = 10$ dB, and $\bar{\gamma}_B = 20$ dB.

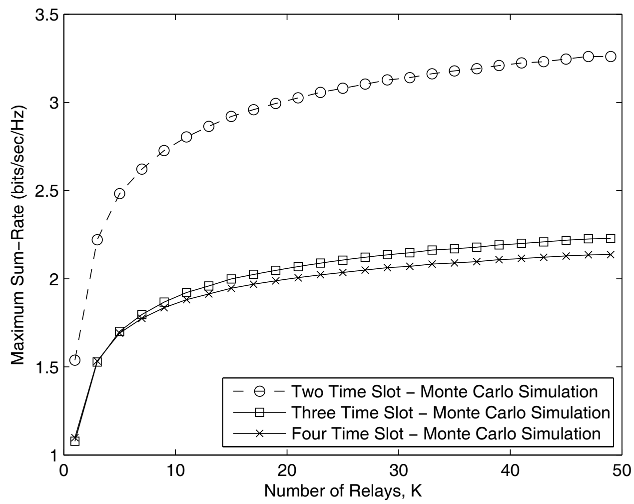

Fig. 10. Maximum sum-rate of the two, three and four time slot opportunistic relay scheme with $\bar{\gamma}_A = \bar{\gamma}_r = 10$ dB, and $\bar{\gamma}_B = 20$ dB.

Corollary 5: At high SNR and sufficiently large K, the difference between the maximum sum-rate of the two and four time slot transmission schemes is given by

$$R_{2\text{TS}}^{\text{multi}} - R_{4\text{TS}}^{\text{multi}} = \frac{1}{4}\log_2\left(\frac{\left(\sqrt{2}+\sqrt{\frac{\bar{\gamma}_r}{\bar{\gamma}}}\right)^2 \bar{\gamma}_r}{\left(1+\sqrt{\frac{\bar{\gamma}_r}{\bar{\gamma}}+1}\right)^4}\right) \tag{53}$$

where $\bar{\gamma} = \frac{P_A}{\sigma_A^2} = \frac{P_B}{\sigma_B^2}$.

Proof: The proof follows by substituting (26) and (28) into (52) for the two and four time slot transmission scheme respectively, and taking the difference. ■

From (53), we see that if $\bar{\gamma} >> \bar{\gamma}_r$, the maximum sum-rate of the two time slot PNC scheme is greater than the four time slot transmission scheme if $\bar{\gamma}_r > 8$.

C. Opportunistic Relay Selection

Fig. 9 shows a plot of the sum-BER vs. the number of relays for two, three and four time slot transmission schemes. We see a significant decrease in the sum-BER as more relays are allowed to be opportunistically chosen. We also see that the four time slot transmission scheme performs significantly better than the two time slot PNC scheme, as predicted by our analytical results. Furthermore, as with the maximum sum-rate, the performance of the three time slot PNC scheme lies between the two and four time slot transmission scheme.

Fig. 10 shows a plot of the maximum sum-rate vs. the number of relays for the two, three and four time slot transmission schemes. We see a significant increase in the maximum sum-rate as more relays are allowed to be opportunistically chosen. We also see that the two time slot PNC scheme performs significantly better than the four time slot transmission scheme, as predicted by our analytical results. Furthermore, we see that the performance of the three time slot PNC scheme lies between the two and four time slot transmission scheme. Note that although the three time slot scheme seems to only perform marginally better than the two time slot scheme, the relative performance would be greater for different source and relay power combinations.

VI. Conclusion

In this paper, we first considered a two-way relay network where two source nodes communicate to each other through a relay node using an AF protocol. We considered communication over either two, three or four time slots. Through analytical and numerical analysis, we have shown that the two and four time slot transmission schemes may outperform the other, depending on different practical scenarios, and whether the performance metric is the maximum sum-rate or sum-BER. In particular, we have shown that at high SNR, the two time slot PNC scheme performs *worse* than the four time slot transmission scheme in terms of sum-BER for sufficiently different source powers, while the two time slot PNC scheme performs *better* than the four time slot transmission scheme in terms of maximum sum-rate. We considered the three time slot PNC scheme, which we showed achieves a performance which either lies between, or exceeds, the performance of the two and four time slot transmission scheme, and thus offers a good compromise. Finally, we considered an opportunistic relaying scheme with K relays, and showed that the use of additional relays can significantly increase system performance, and offers a diversity order K times the diversity order when only one relay is used.

Appendix

A. Proof of Theorem 1

In order to derive performance measures for the three transmission schemes, it is convenient to first express the received SNR in a general form as

$$\gamma = \frac{\beta_{A,\tau\text{TS}}XY}{\theta_{A,\tau\text{TS}}Y + \phi_{A,\tau\text{TS}}X + \zeta} \tag{54}$$

where X and Y are exponentially distributed random variables, $\beta_{A,\tau\text{TS}}$, $\theta_{A,\tau\text{TS}}$ and $\phi_{A,\tau\text{TS}}$ are constants related to the transmit SNR at the source and relay nodes. A general form outage probability expression can thus be written as

$$\begin{aligned}&\Pr\left(\frac{\beta_{A,\tau\mathrm{TS}}XY}{\theta_{A,\tau\mathrm{TS}}Y+\phi_{A,\tau\mathrm{TS}}X+\zeta}\le z\right)\\&=\Pr\left(X\le\frac{z(\theta_{A,\tau\mathrm{TS}}Y+\zeta)}{\beta_{A,\tau\mathrm{TS}}Y-z\phi_{A,\tau\mathrm{TS}}}\right)\mathbf{1}_{Y\ge\frac{z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}+\mathbf{1}_{Y<\frac{z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}\\&=1-e^{-\frac{z(\theta_{A,\tau\mathrm{TS}}Y+\zeta)}{\beta_{A,\tau\mathrm{TS}}Y-z\phi_{A,\tau\mathrm{TS}}}}\mathbf{1}_{Y\ge\frac{z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}\end{aligned}\tag{55}$$

where $\mathbf{1}_{Y\ge X}=1$ if $Y\ge X$ and 0 otherwise. Integrating out Y, we have

$$\begin{aligned}F_\gamma(z)&=1-\int_{\frac{z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}^{\infty}e^{-\frac{z(\theta_{A,\tau\mathrm{TS}}y+\zeta)}{\beta_{A,\tau\mathrm{TS}}y-z\phi_{A,\tau\mathrm{TS}}}}e^{-y}\mathrm{d}y\\&=1-\frac{1}{\beta_{A,\tau\mathrm{TS}}}\int_0^\infty e^{-\frac{z\left(\theta_{A,\tau\mathrm{TS}}\left(\frac{w+z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}\right)+\zeta\right)}{w}}e^{-\frac{w+z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}\mathrm{d}w\\&=1-\frac{e^{-\frac{z\phi_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}e^{-\frac{z\theta_{A,\tau\mathrm{TS}}}{\beta_{A,\tau\mathrm{TS}}}}}{\beta_{A,\tau\mathrm{TS}}}\int_0^\infty e^{-\frac{z\left(z\theta_{A,\tau\mathrm{TS}}\phi_{A,\tau\mathrm{TS}}+\beta_{A,\tau\mathrm{TS}}\zeta\right)}{\beta_{A,\tau\mathrm{TS}}w}}\\&\quad\times e^{-\frac{w}{\beta_{A,\tau\mathrm{TS}}}}\mathrm{d}w\,.\end{aligned}\tag{56}$$

The result follows by solving the integral using identities in [24].

B. Proof of Lemma 1

A general form for the received SNR using the CA-AF gain is given by

$$Z=\frac{\beta_{A,\tau\mathrm{TS}}XY}{\theta_{A,\tau\mathrm{TS}}Y+\phi_{A,\tau\mathrm{TS}}X}=\frac{\beta_{A,\tau\mathrm{TS}}}{\theta_{A,\tau\mathrm{TS}}\phi_{A,\tau\mathrm{TS}}}W\,.\tag{57}$$

To proceed, we note that W is the received SNR of a two hop network using the CA-AF gain, with transmit SNR at the source and relay given by $\theta_{A,\tau\mathrm{TS}}$ and $\phi_{A,\tau\mathrm{TS}}$. Now at high $\theta_{A,\tau\mathrm{TS}}$ and $\phi_{A,\tau\mathrm{TS}}$, we note that W can be closely approximated by $W_{\min}=\min\left(\theta_{A,\tau\mathrm{TS}}Y,\phi_{A,\tau\mathrm{TS}}X\right)$. Since $\theta_{A,\tau\mathrm{TS}}Y$ and $\phi_{A,\tau\mathrm{TS}}X$ are both exponential distributed random variables with parameters $\theta_{A,\tau\mathrm{TS}}$ and $\phi_{A,\tau\mathrm{TS}}$ respectively, the c.d.f. of $W_{\min}$ is given by

$$F_{W_{\min}}(w)=1-e^{-w\left(\frac{1}{\theta_{A,\tau\mathrm{TS}}}+\frac{1}{\phi_{A,\tau\mathrm{TS}}}\right)}\tag{58}$$

and thus by using (57) and (58), we have

$$F_Z(z)\le F_{Z_{\min}}(z)=1-e^{-\frac{z\left(\theta_{A,\tau\mathrm{TS}}+\phi_{A,\tau\mathrm{TS}}\right)}{\beta_{A,\tau\mathrm{TS}}}}\,.\tag{59}$$

C. Proof of Lemma 7

We first give the following lemma:

Lemma 8: For sufficiently large K,

$$R_{\tau\mathrm{TS}}^{\mathrm{multi}}=\log_2\left(1+\frac{\rho}{N_t}F_X^{-1}\left(\frac{K}{K+1}\right)\right)\,.\tag{60}$$

Proof: The proof follows by applying a general result from order statistics [25] to $R_{\tau\mathrm{TS}}$, and performing some simple algebraic manipulation. ∎

Lemma 8 implies that for large K, we need only consider the outage probability in (29) in the high γ_{th} regime. By using the asymptotic expansion for $K_1(\cdot)$ in [19], the outage probability at high γ_{th} is given by

$$\begin{aligned}F_\gamma^\infty(\gamma_{\mathrm{th}})=1-&\sqrt{\frac{\pi\gamma_{\mathrm{th}}\sqrt{\theta_{A,\tau\mathrm{TS}}\phi_{A,\tau\mathrm{TS}}}}{\beta_{A,\tau\mathrm{TS}}}}\\&\times e^{-\frac{\gamma_{\mathrm{th}}\left(\sqrt{\phi_{A,\tau\mathrm{TS}}}+\sqrt{\theta_{A,\tau\mathrm{TS}}}\right)^2}{\beta_{A,\tau\mathrm{TS}}}}\,.\end{aligned}\tag{61}$$

The result now follows by using the approach to find $F^{-1}(\cdot)$ in [20], and substituting the resultant expression in (60), followed by some algebraic manipulations.

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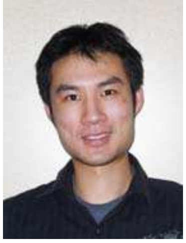

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