# Opportunistic Scheduling and Spectrum Reuse in Relay-Based Cellular Networks

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***Abstract*—In order to understand the key merits of multiuser diversity, opportunistic scheduling and spectrum reuse techniques in downlink relay-based cellular networks, this paper analyzes the spectral efficiency performance over a fading multihop broadcast channel (MBC) composed of a base station (BS), a relay station (RS), $U$ far users and $V$ near users in the asymptotic regime of large number of users. Using tools from extreme-value theory, we characterize the average spectral efficiency of the MBC as a function of the number of users and physical channel parameters. Our analysis yields very accurate formulas even for moderately low values of $U$ and $V$, specified in detail and verified (via Monte Carlo simulations) for the case of Rayleigh fading. Next, we consider a relay-based broadband multi-cellular network in the downlink mode; with special focus on orthogonal frequency-division multiple-access (OFDMA) resource allocation and investigate the spectral efficiency performance of opportunistic multiuser scheduling and spectrum reuse techniques based on capacity analysis and system-level simulations. This empirical study validates our analytical insights and helps to further identify design tradeoffs associated with spectrum reuse, interference management and multiuser diversity techniques in relay-based cellular networks.**

***Index Terms*—Radio resource management, cellular networks, wide area networks, multihop relaying, multiuser diversity, opportunistic scheduling, spectrum reuse, orthogonal frequency division multiple access (OFDMA), extreme value theory.**

## I. INTRODUCTION

THE rapid deployment of broadband wireless access networks over large coverage areas (e.g., wide-area networks (WANs)) calls for the investigation of low-cost and high-performance infrastructure technologies. In this context, a *cellular multihop/mesh architecture* (e.g., IEEE 802.16j/m systems) could provide a leverage for better capacity, coverage and reliability without requiring significant infrastructure deployment costs [1]. An example architecture is depicted in Fig. 1, where the role of the additional infrastructure deployment points is to serve as wireless relay terminals (labeled as RS, i.e., relay station) for the data to be *routed* between the wired infrastructure devices (labeled as BS, i.e., base station) and end users (labeled as MS, i.e., mobile station) and thereby to enhance the quality of end-to-end communication.

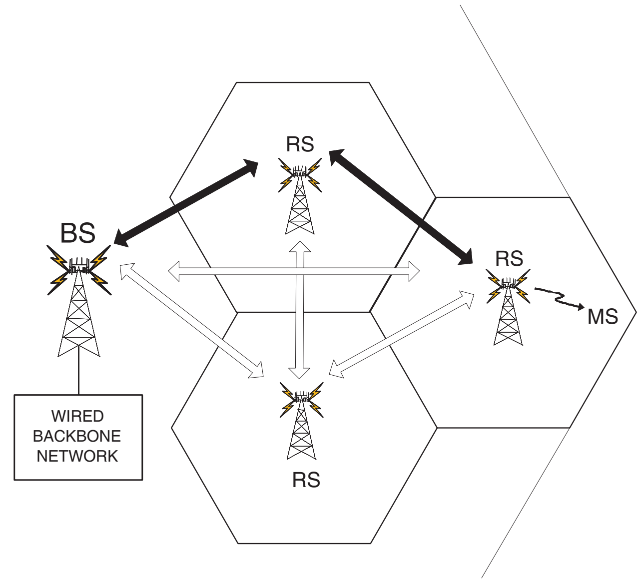


Fig. 1. Relay-assisted cellular multihop wireless network model.

For fixed portable applications, where radio channels are slowly varying, multiple access methods based on *opportunistic* (i.e., channel-aware) scheduling mechanisms take advantage of independent variations in users' link quality and allocate resources such that the user with the best channel quality is served at any given time or frequency (could be subject to certain quality of service (QoS) constraints such as fairness and delay). It has been shown by the pioneering works [2]-[4] that the sum capacity under such opportunistic scheduling algorithms increases with the number of users, yielding *multiuser diversity* gains by exploiting the time and frequency selectivity of wireless channels as well as the independent channel variations across users.

Current and evolving standards for broadband wireless systems (e.g., IEEE 802.16, 3G LTE, LTE Advanced, etc.) are adopting orthogonal-frequency division multiple access (OFDMA) as the resource allocation policy, in which the available time and frequency resources over each wireless link are orthogonally allocated across users, avoiding inter-user interference and impairments due to multipath fading. An intrinsic advantage of OFDMA over other multiple access methods is its flexibility; which allows it to realize multiuser diversity gains over both time and frequency domains through opportunistic scheduling mechanisms [5]-[7]. To further maxi-

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mize spectral efficiency, it is also preferable to *reuse* spectrum opportunistically under OFDMA resource allocation; in fact future cellular networks are evolving toward frequency reuse of one, i.e. all cells/sectors operate on the same frequency channel.

*Contributions:* While multiuser diversity, opportunistic scheduling and spectrum reuse concepts over traditional (point-to-multipoint and multipoint-to-point) cellular systems are now well understood ([8], Chapter 6), it is not yet clear how to extend these concepts to relay-assisted multiuser communication settings, particularly to relay-based cellular OFDMA networks. The goal of this paper is to understand the key merits and address certain open issues on the design and analysis of opportunistic scheduling, spectrum reuse and interference management algorithms over relay-based cellular networks. For this purpose, we focus on relay-assisted multiuser communication in the broadcast (downlink) mode and initially consider an isolated single-cell system (i.e., with no co-channel interference) to which we refer as the *multihop broadcast channel* (MBC), with one BS, one fixed RS, $U$ far users and $V$ near users.

We analyze the spectral efficiency of the opportunistic scheduling and spectrum reuse algorithms over the MBC in the asymptotic regime of large number of users. In this regime, merely investigating the asymptotic capacity scaling as $U, V \to \infty$ (which was the standard approach in many works on traditional cellular systems) limits the scope of the probabilistic analysis and furthermore does not provide accurate insights on the large system behavior of the MBC as many statistical fading models relevant in practice start to break down and become unrealistic for very large numbers of users. Instead, our approach involves using tools from *extreme-value theory* to approximate the *distribution* of spectral efficiency, which is tight provided that $U$ and $V$ are large enough. Based on this framework, we investigate the average spectral efficiency of the MBC as a function of the number of users and physical channel signal-to-noise ratios (SNRs). Our analysis results in very accurate formulas even for moderately low values of $U$ and $V$, specified in detail and verified (via Monte Carlo simulations) for the case of Rayleigh fading.

In addition to the asymptotic spectral efficiency analysis over the simple MBC model, this paper also presents capacity analysis and simulation results on the *system-level performance* of opportunistic scheduling and spectrum reuse techniques in a relay-based multi-cellular communication environment. Our system-level performance analysis accounts for the presence of realistic broadband channel propagation conditions, co-channel interference and OFDMA modulation. In particular, we consider a multihop cellular network model utilizing from single-hop transmission protocols for serving near users (i.e., cell-center users) and relay-assisted two-hop transmission protocols for serving far users (i.e., cell-edge users) with varying degrees of reuse (of time and frequency resources) involving orthogonalized or simultaneous transmissions by the BS and RSs in each cell. This empirical study validates some of the analytical insights developed by our extreme-value theoretic framework (which focuses on the single cell setting with no co-channel interference) and draws further insights on the key merits of spectrum reuse, interference management and multiuser diversity techniques in relay-based cellular networks. We have reported our results earlier in [9]-[10]. Finally, the applications of opportunistic communication principles to relay-assisted wireless networks were investigated in various other contexts in [11]-[14].

The rest of this paper is organized as follows. We discuss the system model and assumptions for the relay-assisted downlink multiuser communication scenario in Section II. The extreme-value theoretic spectral efficiency analysis on the opportunistic scheduling and spectrum reuse algorithms over the MBC model is presented in Section III. Extensions to multi-cellular broadband OFDMA networks are provided in Section IV based on capacity analysis and simulation results on the system-level performance. Finally, we conclude the paper in Section V.

## II. System Model and Assumptions

### A. Protocol Assumptions

We focus on relay-assisted multiuser communication in the broadcast (downlink) mode and initially consider an isolated single-cell system under the MBC model (i.e., with no co-channel interference), with one BS, one fixed RS, $U + V$ users which receive information from the BS, as depicted in Fig. 2. The users are divided into two categories: $U$ far users, indexed by $u = 1, ..., U$, with poor quality links to the BS, and $V$ near users, indexed by $v = 1, ..., V$, with high quality links to the BS. The role of the RS is to enhance the end-to-end link quality for the far users in terms of capacity, coverage and reliability using multihop routing techniques [1], and its presence allows the BS to communicate with the far users over a two-hop route such that the BS sends data to the RS over a high-capacity wireless backhaul link and the RS decodes, re-encodes and forwards the data to the far users with possible interference from the BS over the second hop (i.e., a decode-and-forward based relaying protocol is assumed [15]). Meanwhile, the near users receive downlink data from the BS directly (with no help from the RS) over the same bandwidth with possible interference from the RS. We assume a time-division based (half duplex) relaying constraint on the multihop routing protocols, which is due to the practical limitation that terminals can often not transmit and receive at the same time. We refer to this communication model as the *multihop broadcast channel* (MBC); which is a special case of the more general broadcast relay channel [16], [17].

### B. Channel Model Assumptions

We assume that all the links over the MBC are corrupted by additive white Gaussian noise (AWGN). Furthermore, the links between the BS and users are assumed to be under frequency-flat multiplicative fading i.i.d. across users, with complex channel gains $\{h_{F,u}\}_{u=1}^{U}$ for $U$ far users and $\{h_{N,v}\}_{v=1}^{V}$ for $V$ near users[1], where $h_{F,u} \in \mathbb{C}$ and $h_{N,v} \in \mathbb{C}$ are complex-valued random variables drawn from an arbitrary continuous distribution $F_h$ with $\mathbb{E}[|h_{F,u}|^2] = \mathbb{E}[|h_{N,v}|^2] = 1, \forall u, v$. The average received SNR for the link between the BS

[1]Subscript $F$ stands for *far* and $N$ stands for *near*. Superscript $b$ stands for *base* station and $r$ stands for *relay* station.

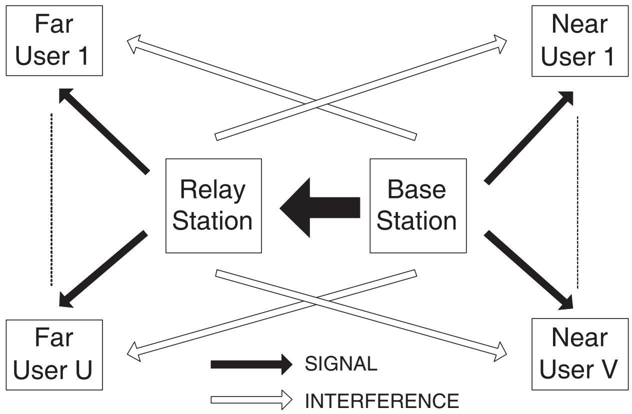


Fig. 2. Multihop broadcast channel (MBC) model (downlink).

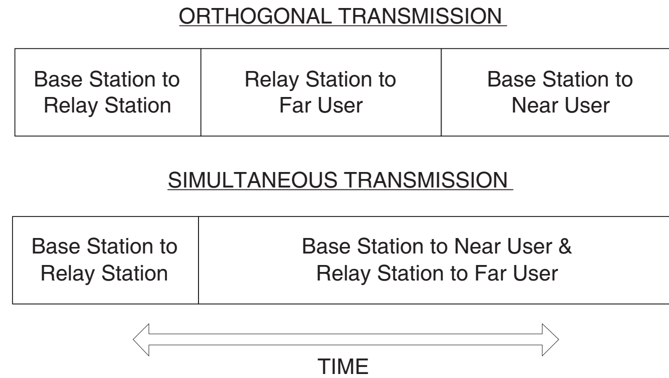


Fig. 3. Multihop time allocation policies based on orthogonal and simultaneous transmission protocols.

and each far user equals $\mathsf{SNR}_F^{(b)}$ and the average received SNR for the link between the BS and each near user equals $\mathsf{SNR}_N^{(b)}$. Analogously, the links between the RS and users are assumed to be under frequency-flat multiplicative fading i.i.d. across users, with complex channel gains $\{g_{F,u}\}_{u=1}^U$ for $U$ far users and $\{g_{N,v}\}_{v=1}^V$ for $V$ near users, where $g_{F,u} \in \mathbb{C}$ and $g_{N,v} \in \mathbb{C}$ are complex-valued random variables drawn from an arbitrary continuous distribution $F_g$ with $\mathbb{E}[|g_{F,u}|^2] = \mathbb{E}[|g_{N,v}|^2] = 1$, $\forall u, v$. The average received SNR for the link between the RS and each far user equals $\mathsf{SNR}_F^{(r)}$ and the average received SNR for the link between the RS and each near user equals $\mathsf{SNR}_N^{(r)}$. The set of channel gains $\{h_{F,u}\}_{u=1}^U$, $\{h_{N,v}\}_{v=1}^V$, $\{g_{F,u}\}_{u=1}^U$ and $\{g_{N,v}\}_{v=1}^V$ are independent. It is assumed that the cellular backhaul link between the BS and RS is an AWGN line-of-sight (LOS) connection with received SNR equal to $\mathsf{SNR}_B$. The fading states over the multiuser links remain constant during the transmission of a codeword and the channel coherence time is much larger than the coding blocklength (slow fading assumption). Due to slow fading, the BS and RS can obtain reliable channel quality information from the users. In particular, each far user can measure its link quality to the RS (e.g., signal-to-interference-plus-noise ratio (SINR)) and feed back this information to the RS. Similarly, the BS can collect link quality information from the near users.

### C. Spectrum Reuse Policies over the MBC

Consider the MBC resource allocation problem such that $U$ far users and $V$ near users are to be assigned time slots for reception over a common bandwidth. This problem involves transmissions over three types of links: (i) $L_B$: The wireless backhaul link between the BS and RS, (ii) $L_F$: The link between the RS and far users, and (iii) $L_N$: The link between the BS and near users. We assign positive time-sharing coefficients $\beta_B$, $\beta_F$ and $\beta_N$ to links $L_B$, $L_F$ and $L_N$ such that $\beta_B + \beta_F + \beta_N = 1$ and define the following reuse policies as depicted in Fig. 3:

*a) Orthogonal transmission (no spectrum reuse):* The links $L_B$, $L_F$ and $L_N$ are active over different time resources with the corresponding time-sharing constants $\beta_B$, $\beta_F$ and $\beta_N$, respectively. Consequently, the discrete-time complex baseband input-output relation to represent the received signal for far user $u$ is given by

$$y_{F,u} = \sqrt{\mathsf{SNR}_F^{(r)}}\, g_{F,u}\, s^{(r)} + z_{F,u},$$

and that to represent the received signal at near user $v$ is given by

$$y_{N,v} = \sqrt{\mathsf{SNR}_N^{(b)}}\, h_{N,v}\, s^{(b)} + z_{N,v},$$

where $s^{(r)} \in \mathbb{C}$ and $s^{(b)} \in \mathbb{C}$ are the data input signals transmitted from the RS and BS, respectively, satisfying the average power constraint $\mathbb{E}[|s^{(r)}|^2] = \mathbb{E}[|s^{(b)}|^2] = 1$, and $z_{F,u} \in \mathbb{C}$ and $z_{N,v} \in \mathbb{C}$ are temporally white circularly symmetric complex zero-mean unit-variance AWGN values observed at far user $u$ and near user $v$, respectively.

*b) Simultaneous transmission (spectrum reuse):* The link $L_B$ is active over $\beta_B$ fraction of the time, while the links $L_F$ and $L_N$ are simultaneously active over $\beta_{NF} = \beta_F + \beta_N$ fraction of the time ($\beta_B + \beta_{NF} = 1$). Consequently, the discrete-time complex baseband input-output relation to represent the received signal for far user $u$ is given by

$$y_{F,u} = \sqrt{\mathsf{SNR}_F^{(r)}}\, g_{F,u}\, s^{(r)} + \sqrt{\mathsf{SNR}_F^{(b)}}\, h_{F,u}\, s^{(b)} + z_{F,u},$$

and that to represent the received signal at near user $v$ is given by

$$y_{N,v} = \sqrt{\mathsf{SNR}_N^{(b)}}\, h_{N,v}\, s^{(b)} + \sqrt{\mathsf{SNR}_N^{(r)}}\, g_{N,v}\, s^{(r)} + z_{N,v}.$$

## III. Spectral Efficiency Analysis of Opportunistic Scheduling in MBCs

In this section, we study the spectral efficiency performance of opportunistic scheduling and spectrum reuse policies over the MBC in the asymptotic regime of large $U$ and $V$ based on an analytical framework that utilizes extreme-value theory.

### A. Extreme-Value Theoretic Preliminaries

Let $\xi_1, \xi_2, ..., \xi_M$ be independently and identically distributed (i.i.d.) random variables drawn from a common cumulative distribution function (CDF) $F(x)$ and denote the maximum of the sequence by $X_M = \max_{m=1,...,M} \xi_m$. If there exist sequences of constants $a_M > 0$, $b_M$, and some non-degenerate distribution function $\mu$ such that $(X_M - b_M)/a_M$

(weakly) converges in distribution to $\mu$ as $M \to \infty$, then $\mu$ belongs to one of the three families of extreme-value distributions: Frechet, Weibull and Gumbel distributions [18]-[20]. The distribution function of $\xi_m$, $F$, determines the exact limiting distribution.

In this paper, we shall mainly be concerned with the case for which wireless fading results in received channel power distributions on each transmit-receive pair link following the Type I extreme-value distribution, where $\mu(x) = \exp(-\exp(-x))$; and this distribution function is known as the Gumbel distribution. Assuming absolutely continuous parent distributions $F$ with density $f$, and that there exists a real number $x_1$ and $x_2 \leq \infty$ with $x_2 = \sup\{x : F(x) < 1\}$ such that, for all $x_1 \leq x < x_2$, $f$ has a negative derivative $f'$ and $f(x) = 0$ for $x \geq x_2$, we can use the following sufficient condition to determine if the parent distribution function $F(x)$ belongs to the Type I domain of attraction (Theorem 1.6.1 in [18]):

$$\lim_{x \to x_2} \frac{f'(x)(1-F(x))}{f^2(x)} = -1 \quad (1)$$

Many fading distributions, e.g. Rayleigh, Ricean, lognormal, could be given as examples leading to Type I convergence. We can determine the normalizing constants $a_M$ and $b_M$ for Type I convergence, by solving for $b_M$ in $1 - F(b_M) = 1/M$ and setting $a_M = \eta(b_M)$, where $\eta$ is the reciprocal hazard function $\eta(x) = (1 - F(x))/f(x)$.

Defining the spectral efficiency function as $C(x) = \log_2(1 + x)$, the following lemma presents a key extreme-value theoretic result to be utilized in the forthcoming capacity analysis.

**Lemma 1:** *Let $\xi_1, \xi_2, ..., \xi_M$ be i.i.d. random variables and assume that there exist sequences of constants $a_M$, $b_M$ (the choices depend on $F$) such that $X_M = \max_{m=1,...,M} \xi_m$ satisfies*

$$\mathbb{P}\left(\frac{X_M - b_M}{a_M} \leq x\right) \to \mu(x) \quad \text{as } M \to \infty \quad (2)$$

*for some limiting extreme-value distribution $\mu$. Now, let $\upsilon_m = C(\mathsf{SNR}\,\xi_m)$ and $Y_M = \max_m \upsilon_m$ for a given constant $\mathsf{SNR} > 0$. Let an event $\mathcal{A}$ be defined in the probability space of random variable $\Theta$ that follows distribution $\mu$, i.e., $\mathcal{A} \subseteq \Omega$, where $\Omega$ is the sample space of $\Theta$. In the regime of large $M$, the expected value of $Y_M$ in $\mathcal{A}$, defined as $\mathsf{Y}_M = \mathbb{E}[Y_M \mathbb{I}(\mathcal{A})]$, can then be approximated as a function of sequences of constants $c_M$ and $d_M$ and event $\mathcal{A}$ as* [2]

$$\mathsf{Y}_M(c_M, d_M, \mathcal{A}) \approx d_M \mathbb{P}(\mathcal{A}) + \sum_{n=1}^{\infty} \Psi_n(\mathcal{A}) \frac{(-1)^{n-1}}{n} \log_2(e)\,(c_M)^n \quad (3)$$

*where $\Psi_n(\mathcal{A}) = \mathbb{E}[\Theta^n \mathbb{I}(\mathcal{A})]$, which can be written in case of Type I convergence such that $\Theta$ follows the Gumbel distribution (i.e., $\mu(x) = \exp(-\exp(-x))$ with $\Omega = (-\infty, \infty)$) as*

$$\Psi_n(\mathcal{A}) = \int_{\tau(\mathcal{A})} (-1)^n [\log(x)]^n \exp(-x) dx, \quad (4)$$

*where set $\tau(\mathcal{A}) \subseteq [0, \infty)$ is defined as $\tau(\mathcal{A}) = \{x : -\log(x) \in \mathcal{A}\}$, the sequence $c_M$ should satisfy $c_M \to 0$ as $M \to \infty$, and $c_M$ and $d_M$ can be related to $a_M$ and $b_M$ as*

$$c_M = \frac{\mathsf{SNR}\,a_M}{1 + \mathsf{SNR}\,b_M}, \qquad d_M = \log_2(1 + \mathsf{SNR}\,b_M) \quad (5)$$

**Proof:** We observe that the spectral efficiency as a function of the received channel power is of the form $f(y) = \log_2(1 + \alpha y)$ for any constant $\alpha > 0$ and therefore is a monotonically increasing function. Exploiting such monotone transformation from received channel powers to spectral efficiencies, we can write $Y_M = \max_m \upsilon_m = \max_m C(\mathsf{SNR}\,\xi_m) = C(\mathsf{SNR}\max_m \xi_m) = C(\mathsf{SNR}\,X_M)$. Given $X_M = a_M \Theta_M + b_M$ such that $\mathbb{P}(\Theta_M \leq x) \to \mu(x)$ as $M \to \infty$ due to (2), we have $\Theta_M \to \Theta$ in distribution as $M \to \infty$, suggesting that $X_M \approx a_M \Theta + b_M$ is a tight approximation for the regime of large $M$, especially if the weak convergence is sufficiently fast. The Taylor series expansion of $Y_M = C(\mathsf{SNR}\,X_M) \approx \log_2(1 + \mathsf{SNR}\,(a_M \Theta + b_M))$ around $\Theta_M = 0$ (also known as Maclaurin series expansion) is given by (the fact that $c_M \to 0$ as $M \to \infty$ ensures that this expansion is valid)

$$Y_M \approx \log_2(1 + \mathsf{SNR}\,b_M) + \sum_{n=1}^{\infty} \Theta^n \frac{(-1)^{n-1}}{n} \log_2(e) \left(\frac{\mathsf{SNR}\,a_M}{1 + \mathsf{SNR}\,b_M}\right)^n \quad (6)$$

from which we obtain $c_M$ and $d_M$ as in (5). The rest of the proof to arrive at (3) can now be easily completed by multiplying (6) by $\mathbb{I}(\mathcal{A})$, taking a straightforward expectation of the resulting product and observing that if $\Theta$ follows the Gumbel distribution, then we can write $\Theta = -\log(\widehat{\Theta})$, where $\widehat{\Theta}$ has exponential distribution with unit mean. □

**Remarks:** Lemma 1 and associated result in (3) constitute an approximation to the expected value of $Y_M$ in its the most general form (tightness to be verified in Section III.D). In the case of $\mathcal{A} = \Omega = (-\infty, \infty)$, $\mathsf{Y}_M$ is given by

$$\mathsf{Y}_M(c_M, d_M, \Omega) \approx d_M + \sum_{n=1}^{\infty} \Psi_n(\Omega) \frac{(-1)^{n-1}}{n} \log_2(e)\,(c_M)^n \quad (7)$$

where $\Psi_n(\Omega) = \mathbb{E}[\Theta^n]$, which can be written in case of Type I convergence such that $\Theta$ follows the Gumbel distribution (i.e., $\mu(x) = \exp(-\exp(-x))$ and $\tau(\Omega) = [0, \infty)$) as

$$\Psi_n(\Omega) = \int_0^{\infty} (-1)^n [\log(x)]^n \exp(-x) dx. \quad (8)$$

For $n = 1, 2$, we can easily show that $\Psi_1(\Omega) = \kappa$, where $\kappa \approx 0.57721566$ is Euler's constant, and $\Psi_2(\Omega) = \kappa^2 + \pi^2/6 \approx 1.9781$. The set of $\{\Psi_n\}$ can in general be conveniently obtained via numerical integration.

## B. Orthogonal Transmissions

We now consider the multiuser scheduling problem such that $U$ far users and $V$ near users are to be assigned time slots for reception over a common bandwidth. The BS and RS employ the maximum SINR (max-SINR) opportunistic scheduling algorithm [8], which always serves the best user with the highest instantaneous rate at any given time/frequency

[2] $\mathbb{I}(\mathcal{A})$ is an indicator random variable such that $\mathbb{I}(\mathcal{A}) = 1$ if event $\mathcal{A}$ is true and $\mathbb{I}(\mathcal{A}) = 0$ if event $\mathcal{A}$ is false.

resource. Our analysis in this section does not address other QoS constraints such as delay and fairness. Accordingly, the RS compares the channel gains $\{|g_{F,u}|^2\}_{u=1}^U$ of the far users and assigns the link $L_F$ for downlink transmission to the far user with the highest instantaneous rate. Analogously, the BS compares the channel gains $\{|h_{N,v}|^2\}_{v=1}^V$ of the near users and assigns the link $L_N$ for downlink transmission to the near user with the highest instantaneous rate.

Assuming Gaussian inputs, i.e., all input signals have the temporally i.i.d. zero-mean circularly symmetric complex Gaussian distribution, the maximum supportable end-to-end spectral efficiency over the MBC achieved by the max-SINR scheduling algorithm in the presence of the orthogonal transmission protocol is given by (in bits per second per Hertz (bps/Hz))

$$\begin{aligned} \mathsf{C}^{\mathrm{ort}} &= \beta_N \max_{v=1,\dots,V} C(\mathsf{SNR}_N^{(b)}|h_{N,v}|^2) \\ &+ \min\left[\beta_B\, C(\mathsf{SNR}_B),\, \beta_F \max_{u=1,\dots,U} C(\mathsf{SNR}_F^{(r)}|g_{F,u}|^2)\right] \end{aligned} \tag{9}$$

In (9), we have two maxima among $U$ and $V$ i.i.d. spectral efficiency random variables, respectively, however the maximum corresponding to the link between the RS and far users is truncated due to the presence of a constant-capacity backhaul link between the BS and RS.

Our goal is to compute the expected value of $\mathsf{C}^{\mathrm{ort}}$ in the limit of large $U$ and $V$, i.e., large number of far and near users, respectively. As the result of Lemma 1 and associated relationships given in (3)-(5) and (7)-(8), in the asymptotic regime of large $U$, $V$, there exist sequences of constants $c_U^{(g)}$, $d_U^{(g)}$, $c_V^{(h)}$, $d_V^{(h)}$ (the choice will depend on the distributions $F_g$ and $F_h$) such that the average spectral efficiency, given by $\Xi^{\mathrm{ort}} = \mathbb{E}\left[\mathsf{C}^{\mathrm{ort}}\right]$ is approximated as

$$\begin{aligned} \Xi^{\mathrm{ort}} &\approx \beta_N\, \Upsilon_V(c_V^{(h)}, d_V^{(h)}, \Omega) \\ &+ \beta_F\, \Upsilon_U(c_U^{(g)}, d_U^{(g)}, \mathcal{A}_{\mathrm{o}}) + \beta_B\, C(\mathsf{SNR}_B)\,(1-\mathbb{P}(\mathcal{A}_{\mathrm{o}})) \end{aligned} \tag{10}$$

where Type I convergence is assumed on the maxima of $\{|g_{F,u}|^2\}_{u=1}^U$ and $\{|h_{N,v}|^2\}_{v=1}^V$ for given sequences of normalizing constants $a_U^{(g)}$, $b_U^{(g)}$, $a_V^{(h)}$ and $b_V^{(h)}$ such that $\max_{u=1,..,U}|g_{F,u}|^2 \approx a_U^{(g)}\Theta + b_U^{(g)}$, and $\max_{v=1,..,V}|h_{N,v}|^2 \approx a_V^{(h)}\Theta + b_V^{(h)}$, where $\Theta$ follows the Gumbel distribution and event $\mathcal{A}_{\mathrm{o}}$ is given by $\mathcal{A}_{\mathrm{o}} = \left\{\Theta : \beta_F\, C(\mathsf{SNR}_F^{(r)}(a_U^{(g)}\Theta + b_U^{(g)})) \le \beta_B\, C(\mathsf{SNR}_B)\right\}$ such that $\mathbb{P}(\mathcal{A}_{\mathrm{o}})$ is given by (11).

*Case of Rayleigh Fading:* Assuming zero-mean circularly symmetric complex Gaussian fading distribution on $F_g$ and $F_h$, the entries in the set of channel gains $\{|g_{F,u}|^2\}_{u=1}^U$ and $\{|h_{N,v}|^2\}_{v=1}^V$ each follows the unit-mean exponential distribution, i.e., $F_{\tilde{g}}(x) = \mathbb{P}\left(|g_{F,u}|^2 \le x\right) = 1 - e^{-x}$, and $F_{\tilde{h}}(x) = \mathbb{P}\left(|h_{N,v}|^2 \le x\right) = 1 - e^{-x}$. It is well established [18] that under Rayleigh fading, $F_{\tilde{g}}$ and $F_{\tilde{h}}$ satisfy (1) and thus belong to the Type I domain of attraction for given sequences of normalizing constants $a_U^{(g)}$, $b_U^{(g)}$, $a_V^{(h)}$ and $b_V^{(h)}$. Setting $1 - F_{\tilde{g}}(b_U^{(g)}) = 1/U$, and $1 - F_{\tilde{h}}(b_V^{(h)}) = 1/V$, we solve for $b_U^{(g)}$ and $b_V^{(h)}$, to arrive at $b_U^{(g)} = \log(U)$ and $b_V^{(h)} = \log(V)$. Moreover, we find that $a_U^{(g)} = a_V^{(h)} = 1$, as a consequence of the observation that the reciprocal hazard functions for the exponential distributions $F_{\tilde{g}}$ and $F_{\tilde{h}}$ are given by $\eta_{\tilde{g}}(x) = \eta_{\tilde{h}}(x) = 1$. Finally, we can use Lemma 1 to explicitly obtain the sequences of normalizing constants $c_U^{(g)}$, $d_U^{(g)}$, $c_V^{(h)}$, $d_V^{(h)}$ from $a_U^{(g)}$, $b_U^{(g)}$, $a_V^{(h)}$ and $b_V^{(h)}$ in the following:

$$c_U^{(g)} = \frac{\mathsf{SNR}_F^{(r)}}{1+\mathsf{SNR}_F^{(r)}\log(U)},\; d_U^{(g)} = \log_2(1+\mathsf{SNR}_F^{(r)}\log(U)) \tag{12}$$

$$c_V^{(h)} = \frac{\mathsf{SNR}_N^{(b)}}{1+\mathsf{SNR}_N^{(b)}\log(V)},\; d_V^{(h)} = \log_2(1+\mathsf{SNR}_N^{(b)}\log(V)) \tag{13}$$

### *C. Simultaneous Transmissions*

The scheduling over the MBC under simultaneous transmissions and max-SINR algorithm occurs as follows: The RS accounts for average received signal-to-noise ratios $\mathsf{SNR}_F^{(r)}$, $\mathsf{SNR}_F^{(b)}$ and channel gains $\{g_{F,u}\}_{u=1}^U$, $\{h_{F,u}\}_{u=1}^U$ to schedule the link $L_F$ for downlink transmission to the far user with the highest instantaneous rate. Similarly, the BS accounts for $\mathsf{SNR}_N^{(b)}$, $\mathsf{SNR}_N^{(r)}$, $\{h_{N,v}\}_{v=1}^V$, $\{g_{N,v}\}_{v=1}^V$ to schedule the link $L_N$ for simultaneous downlink transmission to the near user with the highest instantaneous rate. No scheduling coordination is assumed to be present between the BS and RS to manage the resulting intracell interference.

Assuming Gaussian inputs, the maximum supportable end-to-end spectral efficiency over the MBC achieved by the max-SINR scheduling algorithm in the presence of the simultaneous transmission protocol is given by (in bps/Hz)

$$\begin{aligned} \mathsf{C}^{\mathrm{sim}} &= \beta_{NF} \max_{v=1,\dots,V} C(\mathsf{SINR}_{N,v}) \\ &+ \min\left[\beta_B\, C(\mathsf{SNR}_B),\, \beta_{NF} \max_{u=1,\dots,U} C(\mathsf{SINR}_{F,u})\right], \end{aligned} \tag{14}$$

where $\mathsf{SINR}_{F,u}$ is the SINR for far user $u$ ($u = 1,...,U$) and $\mathsf{SINR}_{N,v}$ is the SINR for near user $v$ ($v = 1,...,V$) given by (the decoders treat interference as Gaussian noise)

$$\mathsf{SINR}_{F,u} = \frac{\mathsf{SNR}_F^{(r)}|g_{F,u}|^2}{\mathsf{SNR}_F^{(b)}|h_{F,u}|^2+1},$$
$$\mathsf{SINR}_{N,v} = \frac{\mathsf{SNR}_N^{(b)}|h_{N,v}|^2}{\mathsf{SNR}_N^{(r)}|g_{N,v}|^2+1},$$

As the result of Lemma 1 and associated relationships given in (3)-(5) and (7)-(8), in the asymptotic regime of large $U$, $V$, there exist sequences of constants $\rho_U^{(f)}$, $\sigma_U^{(f)}$, $\rho_V^{(n)}$, $\sigma_V^{(n)}$ (the choice will depend on the distributions $F_g$ and $F_h$) such that the average spectral efficiency, given by $\Xi^{\mathrm{sim}} = \mathbb{E}\left[\mathsf{C}^{\mathrm{sim}}\right]$ is approximated as

$$\begin{aligned} \Xi^{\mathrm{sim}} &\approx \beta_{NF}\, \Upsilon_V(\rho_V^{(n)}, \sigma_V^{(n)}, \Omega) \\ &+ \beta_{NF}\, \Upsilon_U(\rho_U^{(f)}, \sigma_U^{(f)}, \mathcal{A}_{\mathrm{s}}) + \beta_B\, C(\mathsf{SNR}_B)\,(1-\mathbb{P}(\mathcal{A}_{\mathrm{s}})) \end{aligned} \tag{15}$$

where Type I convergence is assumed on the maxima of $\{\mathsf{SINR}_{F,u}\}_{u=1}^U$ and $\{\mathsf{SINR}_{N,v}\}_{v=1}^V$ for given sequences of normalizing constants $\gamma_U^{(f)}$, $\delta_U^{(f)}$, $\gamma_V^{(n)}$ and

$$\mathbb{P}(\mathcal{A}_{\mathrm{o}}) = \exp\left(-\exp\left(\frac{b_U^{(g)}}{a_U^{(g)}} - \frac{\exp(\beta_B\, C(\mathsf{SNR}_B)/(\log_2(e)\,\beta_F)) - 1}{a_U^{(g)}\,\mathsf{SNR}_F^{(r)}}\right)\right) \tag{11}$$

$$\mathbb{P}(\mathcal{A}_{\mathrm{s}}) = \exp\left(-\exp\left(\frac{\delta_U^{(f)}}{\gamma_U^{(f)}} - \frac{\exp(\beta_B\, C(\mathsf{SNR}_B)/(\log_2(e)\,\beta_{NF})) - 1}{\gamma_U^{(f)}}\right)\right) \tag{16}$$

$\delta_V^{(n)}$ such that $\max_{u=1,\ldots,U}\mathsf{SINR}_{F,u} \approx \gamma_U^{(f)}\Theta + \delta_U^{(f)}$ and $\max_{v=1,\ldots,V}\mathsf{SINR}_{N,v} \approx \gamma_V^{(n)}\Theta + \delta_V^{(n)}$ where $\Theta$ follows the Gumbel distribution and event $\mathcal{A}_{\mathrm{s}}$ is given by $\mathcal{A}_{\mathrm{s}} = \left\{\Theta : \beta_{NF}\, C(\gamma_U^{(f)}\Theta + \delta_U^{(f)}) \le \beta_B\, C(\mathsf{SNR}_B)\right\}$ such that $\mathbb{P}(\mathcal{A}_{\mathrm{s}})$ is given by (16).

*Case of Rayleigh Fading:* Assuming zero-mean circularly symmetric complex Gaussian fading distribution on $F_g$ and $F_h$, the entries in the set of channel gains $\{|g_{F,u}|^2\}_{u=1}^U$, $\{|g_{N,v}|^2\}_{v=1}^V$, $\{|h_{F,u}|^2\}_{u=1}^U$, and $\{|h_{N,v}|^2\}_{v=1}^V$ each follows the unit-mean exponential distribution. Defining the CDF of $\mathsf{SINR}_{F,u}$ by $F_{\mathsf{SINR}_{F,u}}(x) = \mathbb{P}\left(\mathsf{SINR}_{F,u} \le x\right)$, we evaluate $F_{\mathsf{SINR}_{f,u}}$ as in (17)-(20), where (18) follows from the law of total probability and (19) follows from the independence of $g_{F,u}$ and $h_{F,u}$. Similarly, defining the CDF of $\mathsf{SINR}_{N,v}$ by $F_{\mathsf{SINR}_{N,v}}(x) = \mathbb{P}\left(\mathsf{SINR}_{N,v} \le x\right)$, we find that

$$F_{\mathsf{SINR}_{N,v}}(x) = 1 - \left(1 + \frac{\mathsf{SNR}_N^{(r)}}{\mathsf{SNR}_N^{(b)}}\, x\right)^{-1} e^{-x/\mathsf{SNR}_N^{(b)}}, \ \ x \ge 0 \tag{21}$$

It is easy to check that under Rayleigh fading, $F_{\mathsf{SINR}_{F,u}}$ and $F_{\mathsf{SINR}_{N,v}}$ given in (20) and (21) satisfy (1) and thus belong to the Type I domain of attraction for given sequences of normalizing constants $\gamma_U^{(f)}$, $\delta_U^{(f)}$, $\gamma_V^{(n)}$ and $\delta_V^{(n)}$. Setting $1 - F_{\mathsf{SINR}_{F,u}}(\delta_U^{(f)}) = 1/U$ and $1 - F_{\mathsf{SINR}_{N,v}}(\delta_V^{(n)}) = 1/V$, we solve for $\delta_U^{(f)}$ and $\delta_V^{(n)}$ to arrive at

$$\delta_U^{(f)} = \mathsf{SNR}_F^{(r)}\,\mathcal{W}\left(\frac{U}{\mathsf{SNR}_F^{(b)}}\, e^{1/\mathsf{SNR}_F^{(b)}}\right) - \frac{\mathsf{SNR}_F^{(r)}}{\mathsf{SNR}_F^{(b)}},$$

$$\delta_V^{(n)} = \mathsf{SNR}_N^{(b)}\,\mathcal{W}\left(\frac{V}{\mathsf{SNR}_N^{(r)}}\, e^{1/\mathsf{SNR}_N^{(r)}}\right) - \frac{\mathsf{SNR}_N^{(b)}}{\mathsf{SNR}_N^{(r)}},$$

where $\mathcal{W}$ is the LambertW function [21]. Moreover, we can express $\gamma_U^{(f)}$ and $\gamma_V^{(n)}$ in terms of the reciprocal hazard functions as $\gamma_U^{(f)} = \eta_f(\delta_U^{(f)})$ and $\gamma_V^{(n)} = \eta_n(\delta_V^{(n)})$, where $\eta_f(x)$ and $\eta_n(x)$ corresponding to the distributions $F_{\mathsf{SINR}_{f,u}}$ and $F_{\mathsf{SINR}_{n,v}}$, respectively, are given by

$$\eta_f(x) = \frac{\mathsf{SNR}_F^{(r)}\left(\mathsf{SNR}_F^{(r)} + \mathsf{SNR}_F^{(b)}\, x\right)}{\mathsf{SNR}_F^{(r)} + \mathsf{SNR}_F^{(r)}\mathsf{SNR}_F^{(b)} + \mathsf{SNR}_F^{(b)}\, x},$$

$$\eta_n(x) = \frac{\mathsf{SNR}_N^{(b)}\left(\mathsf{SNR}_N^{(b)} + \mathsf{SNR}_N^{(r)}\, x\right)}{\mathsf{SNR}_N^{(b)} + \mathsf{SNR}_N^{(b)}\mathsf{SNR}_N^{(r)} + \mathsf{SNR}_N^{(r)}\, x}.$$

Finally, we can now use Lemma 1 to write the sequences of normalizing constants $\rho_U^{(f)}$, $\sigma_U^{(f)}$, $\rho_V^{(n)}$, $\sigma_V^{(n)}$ as a function of $\gamma_U^{(f)}$, $\delta_U^{(f)}$, $\gamma_V^{(n)}$ and $\delta_V^{(n)}$ in the following:

$$\rho_U^{(f)} = \frac{\gamma_U^{(f)}}{1 + \delta_U^{(f)}}, \qquad \sigma_U^{(f)} = \log_2(1 + \delta_U^{(f)}), \tag{22}$$

$$\rho_V^{(n)} = \frac{\gamma_V^{(n)}}{1 + \delta_V^{(n)}}, \qquad \sigma_V^{(n)} = \log_2(1 + \delta_V^{(n)}). \tag{23}$$

### *D. Numerical Results*

Assuming Rayleigh fading distribution on $F_h$ and $F_g$ and setting $\mathsf{SNR}_B = 1000$, $\mathsf{SNR}_F^{(r)} = \mathsf{SNR}_N^{(b)} = 100$, $\mathsf{SNR}_F^{(b)} = \mathsf{SNR}_N^{(r)} = 1$, we plot in Fig. 4 average spectral efficiency as a function of the number of far / near users (set $U = V$) for orthogonal and simultaneous transmission protocols over the MBC with time-sharing coefficients set to $\beta_B = 0.25$, $\beta_F = 0.25$, $\beta_N = 0.5$. Here, we compare empirically generated average spectral efficiencies $\Xi^{\mathrm{ort}}$ and $\Xi^{\mathrm{sim}}$ (solid curves) with their analytical counterparts in (10) and (15) (dashed curves), with the corresponding normalizing scaling constants specified in (12) and (13) for orthogonal transmissions and in (22) and (23) for simultaneous transmissions. The empirical results are obtained by averaging the expressions in (9) and (14) over a large number of randomly generated fading realizations (based on Monte Carlo simulations). As part of the analytical average spectral efficiency calculations in (10) and (15), a design parameter $N$ was used to bound the upper summation index in (3) and (7), e.g., accordingly $\mathsf{Y}_M$ in (7) was further approximated as

$$\mathsf{Y}_M(c_M, d_M, \Omega) \approx d_M + \sum_{n=1}^{N} \Psi_n(\Omega)\frac{(-1)^{n-1}}{n}\log_2(e)\,(c_M)^n,$$

and the value of this parameter $N$ was optimized for highest accuracy.

From Fig. 4, we validate the accuracy and tightness of the closed-form average spectral efficiency expressions in (10) and (15). In particular, we verify that our analytical results are well in agreement with the empirical results even for moderately low number of users and that higher level of accuracy is achieved with higher $U, V$. In the practically relevant context of a cellular suburban deployment where the number of users per sector typically ranges around 20-30, we find that the approximation error is as low as $1.1\%$ for orthogonal transmission and $1.5\%$ for simultaneous transmission (i.e., when we choose $U = V = 8$). Moreover, we observe that both orthogonal transmission and simultaneous transmission protocols realize multiuser diversity gains from opportunistic scheduling techniques due to the increase of the average spectral efficiency in the number of far / near users. Finally, we find that significant spectral efficiency gains can

$$F_{\mathsf{SINR}_{F,u}}(x) = \mathbb{P}\left(\frac{\mathsf{SNR}_F^{(r)}|g_{F,u}|^2}{\mathsf{SNR}_F^{(b)}|h_{F,u}|^2+1} \le x\right) \quad (17)$$

$$= \int_0^\infty \mathbb{P}\left(\frac{\mathsf{SNR}_F^{(r)}|g_{F,u}|^2}{\mathsf{SNR}_F^{(b)}|h_{F,u}|^2+1} \le x \mid y \le |h_{F,u}|^2 \le y+dy\right) e^{-y}dy \quad (18)$$

$$= \int_0^\infty \mathbb{P}\left(|g_{F,u}|^2 \le \frac{\mathsf{SNR}_F^{(b)}\,y+1}{\mathsf{SNR}_F^{(r)}}\,x\right) e^{-y}dy \quad (19)$$

$$= 1-\left(1+\frac{\mathsf{SNR}_F^{(b)}}{\mathsf{SNR}_F^{(r)}}\,x\right)^{-1} e^{-x/\mathsf{SNR}_F^{(r)}}, \;\; x \ge 0 \quad (20)$$

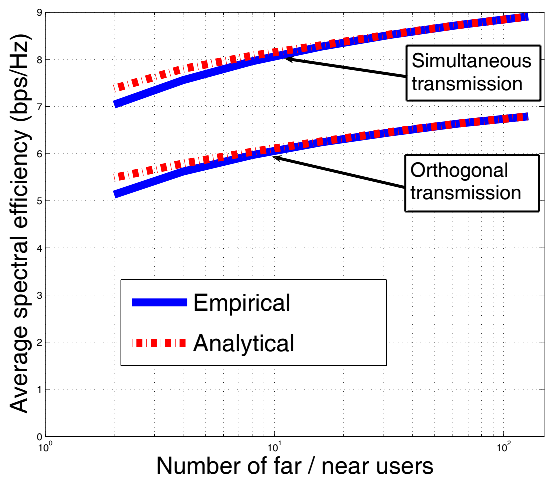


Fig. 4. Average spectral efficiency as a function of the far / near users for the orthogonal and simultaneous transmission protocols in case of Rayleigh fading.

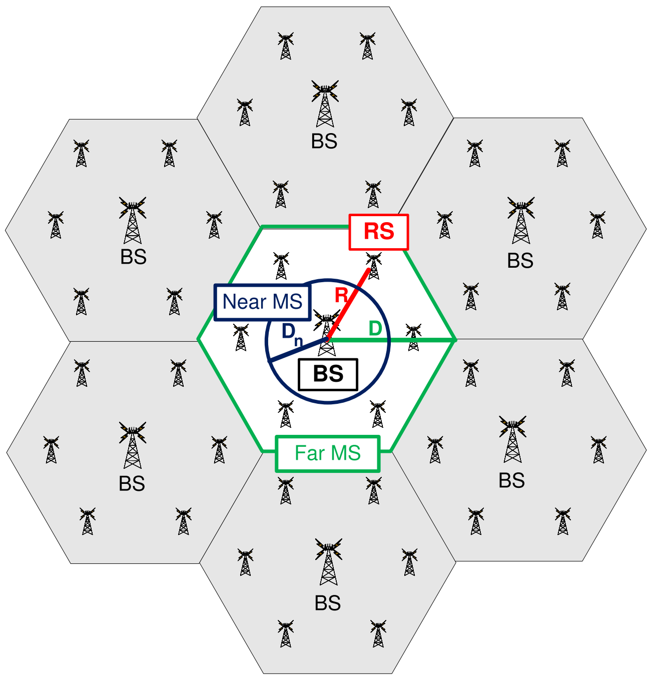


Fig. 5. Relay-based multi-cellular network model.

be achieved through spectrum reuse by the BS and RS based on the superior performance of the simultaneous transmission protocol over the orthogonal transmission protocol.

## IV. Extensions to Multi-Cellular Broadband OFDMA Networks

In order to validate some of the analytical insights developed by our extreme-value theoretic framework in Section III, this section presents capacity analysis and simulation results on the system-level performance of OFDMA-based relay-assisted opportunistic scheduling and spectrum reuse techniques in a multi-cellular communication environment (e.g., wireless metropolitan area networks (WMANs) based on the IEEE 802.16j/m standards [22], [23]) in the presence of realistic broadband channel propagation conditions and co-channel interference [24], [25].

### A. Network Model and Spectrum Reuse Policies

We consider a relay-assisted multi-cellular network, with a center main cell surrounded by a number of interfering cells, where all cells have radius $D$. The model includes a BS placed at the center of each cell and $M$ RSs placed symmetrically with an angular separation of $2\pi/M$ radians at a distance $R$ from the BS ($0 < R < D$). An example network is depicted in Fig. 5 for a single tier (i.e., 6) of interfering cells with $M = 6$. We assume global frequency reuse across all cells, i.e. all cells operate over only one frequency channel to maximize spectrum utilization (frequency reuse 1). The focus of the system-level analysis is on relay-assisted downlink multiuser communication. Furthermore, no sectorization is assumed in all cells, i.e. the BSs and RSs are assumed to possess omnidirectional antennas.

As in the MBC model introduced in Section III, the users in the main cell are divided into two categories: $U$ far users randomly located at the cell edge, with generally poor quality links to the BS and $V$ near users randomly located within a distance $D_n < D$ from the BS (e.g., users at the cell center), with generally high quality links to the BS. The downlink communication between the BS and a given far user takes place over a two-hop route where: (i) the BS sends data to a selected RS over a high capacity wireless backhaul link with co-channel interference from the BSs in the neighboring cells, and (ii) the selected RS decodes and forwards the data over the second hop to a scheduled far user in its coverage

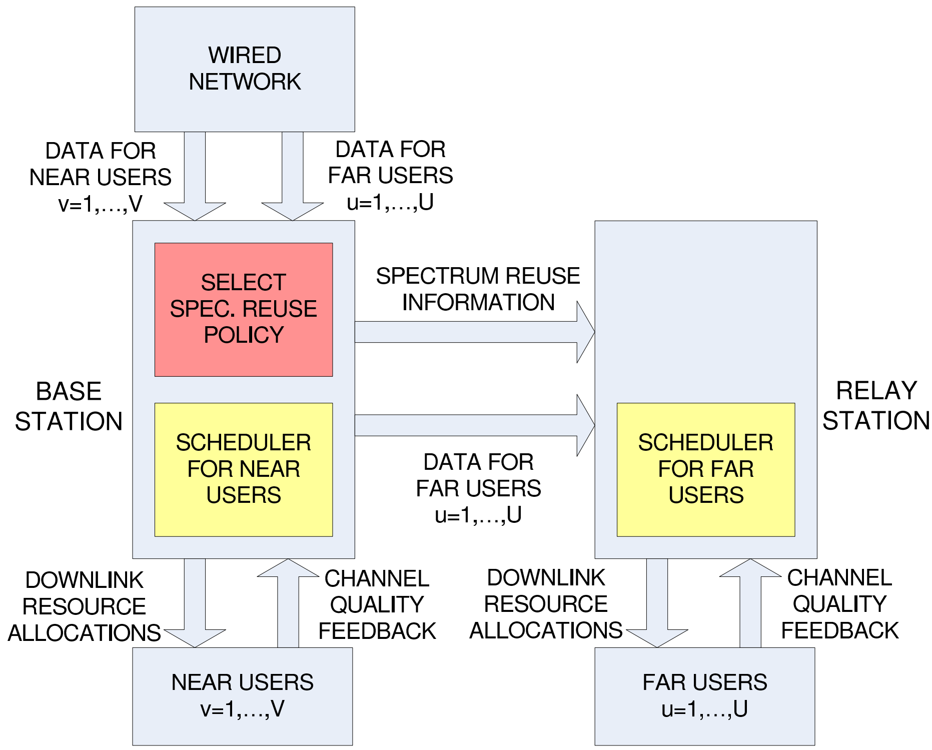


Fig. 6. Downlink resource management framework for opportunistic scheduling and spectrum reuse over the relay-based cellular network.

area with co-channel interference from the BSs and RSs from the neighboring cells and possibly intra-cell interference from the BS and RSs in the main cell. The RS selection, network entry and handoff for a given far user are based on shortest-path principles; i.e., the user gets connected to the closest RS. Moreover, it is assumed that the communication over the BS-RS links are scheduled to occur over a common time-frequency allocation zone such that all RSs can receive all of transmitted information from the BS over the backhaul links. Meanwhile, the near users receive downlink data from the BS directly (with no help from the RSs) over the same bandwidth with interference from the BSs in the neighboring cells and possibly from the RSs in the neighboring cells and main cell. The overall resource management framework for opportunistic scheduling and spectrum reuse over the relay-based cellular network is depicted in Fig. 6. The far and near users inform their respective BS and RSs regarding their dynamically changing link conditions through the available channel quality feedback mechanisms. For all links to the near and far users and all links over the wireless backhaul among BSs and RSs, the channel responses of different links over the relay-assisted multi-cellular network are assumed to be independent.

We observe that the spectrum reuse policies introduced in Section II.C for the MBC and categorized as orthogonal and simultaneous transmission protocols (depending on whether there is spectrum reuse between the BS and RSs in the same cell) are applicable in the context of the relay-based multi-cellular network under our assumption that each far user is served by a single selected RS, i.e., as the BS serves a near user over a given time-frequency allocation zone, whether an RS can also transmit to a far user over the same resources. Additionally, another form of spectrum reuse is relevant our system-level analysis in this section: During any given time in which link $L_F$ is active for downlink transmission, multiple RSs may serve the far users in their respective coverage areas such that each RS transmits to a single scheduled far user over a common set of allocated time-frequency resources. To address such spectrum reuse across multiple RSs within a given cell, we define the *relay reuse factor* $m$, corresponding to the setting in which $M/m$ RS terminals in the each cell simultaneously transmit over the time and frequency resources allocated for link $L_F$. For instance, the case of $m = 1$ implies full relay reuse where all RSs occupy the whole available bandwidth, while the case of $m = M$ implies no relay reuse where the bandwidth is partitioned into $M$ orthogonal blocks and each RS occupies one block when link $L_F$ is active. Example relay frequency reuse policies are depicted in Fig. 7 for the case of $M = 6$. Both orthogonal and simultaneous transmission modes may be able to benefit from relay reuse over the link $L_F$ to improve spectral efficiency, provided that the resultant interference on the far users due to simultaneous transmissions by the RSs does not cause a severe degradation in the received signal quality. It is assumed that the choice between orthogonal transmissions and simultaneous transmissions as well as the selected values of $m$, $\beta_B$, $\beta_F$ and $\beta_N$ are fixed across all cells (i.e. the main cell and interfering cells).

### *B. Multiuser Scheduling Algorithms and Spectral Efficiency Analysis*

We consider multiuser scheduling over $K$ available OFDMA subchannels, indexed by $k = 1, ..., K$. The downlink resource allocation decisions at a given RS for $U$ far

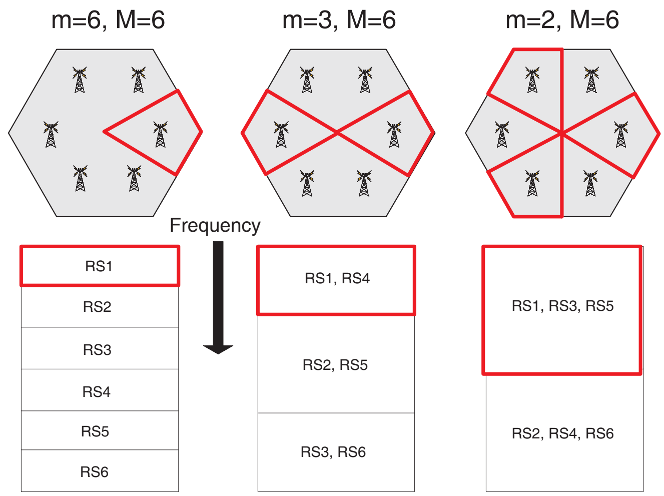


Fig. 7. Example relay frequency reuse policies for $M = 6$.

users and those at the BS for $V$ near users rely on the feedback of SINR estimates from the MSs, and are based on the well-known round-robin, max-SINR (as also analyzed in Section III) and proportional-fair scheduling algorithms [8], [24]. Defining the SINRs of far user $u$ and near user $v$ at time $t$ and subchannel $k$ by $\mathsf{SINR}_{F,u}(t,k)$ and $\mathsf{SINR}_{N,v}(t,k)$, respectively (which depend on the choice between orthogonal transmissions and simultaneous transmissions as well as the selected value of $m$), the max-SINR scheduler of the selected RS assigns subchannel $k$ to far user $\hat{u}_k$ with the largest instantaneous rate at any given time. Thus, the selected far user set $\{\hat{u}_k\}_{k=1}^{K}$ at time $t$ can be expressed as $\hat{u}_k = \arg\max_{u=1,\ldots,U} C(\mathsf{SINR}_{F,u}(t,k))$. Similarly, the max-SINR scheduler of the BS assigns subchannel $k$ at time $t$ to near user $\hat{v}_k$ such that $\hat{v}_k = \arg\max_{v=1,\ldots,V} C(\mathsf{SINR}_{N,v}(t,k))$.

The proportional-fair scheduler at the selected RS keeps track of the average rates $T_u$ for each far user $u$ in an exponentially weighted observation window of length $T_c$, which can be tied to the latency time-scale of the application (i.e., the maximum number of time slots for which an individual user can wait to receive data). The selected far user set $\{\hat{u}_k\}_{k=1}^{K}$ at time $t$ and subchannel $k$ is determined according to the criterion $\hat{u}_k = \arg\max_u \mathsf{C}_{F,u}(t,k)\,/\,T_u(t)$, $k = 1,\ldots,K$, where $\mathsf{C}_{F,u}(t,k)$ is the instantaneous rate served to far user $u$ given by (24) for orthogonal and simultaneous transmission protocols and $T_u(t)$ is the long-term average rate for far user $u$ at time $t$, which is updated based on

$$T_u(t+1) = \left(1 - \frac{1}{T_c}\right) T_u(t) + \frac{1}{K\,T_c} \sum_{k=1}^{K} \mathsf{C}_{F,u}(t,k)\Lambda_u(t,k),$$

where $\mathsf{SINR}_B(t,k)$ is the SINR over the wireless backhaul link between the BS and selected RS at time $t$ and subchannel $k$, factor $M/m$ accounts for relay-based spectrum reuse, and $\Lambda_u(t,k)$ is an indicator random variable that is set to 1 if far user $u$ is scheduled to subchannel $k$ at time $t$ and to 0 otherwise. The operation of the proportional-fair scheduler at the BS occurs similarly to determine the selected near user set $\{\hat{v}_k\}_{k=1}^{K}$ based on $\{\mathsf{SINR}_{N,v}(t,k)\}_{v=1}^{V}$, according to the criterion $\hat{v}_k = \arg\max_v \mathsf{C}_{N,v}(t,k)\,/\,T_v(t)$, where $\mathsf{C}_{N,v}(t,k)$ is the instantaneous rate served to near user $v$ given by $\mathsf{C}_{N,v}(t,k) = \beta_N\, C(\mathsf{SINR}_{N,v}(t,k))$ for orthogonal transmissions and by $\mathsf{C}_{N,v}(t,k) = \beta_{NF}\, C(\mathsf{SINR}_{N,v}(t,k))$ for simultaneous transmissions, and $T_v(t)$ is the long-term average rate for near user $v$ at time $t$, which is updated based on

$$T_v(t+1) = \left(1 - \frac{1}{T_c}\right) T_v(t) + \frac{1}{K\,T_c} \sum_{k=1}^{K} \mathsf{C}_{N,v}(t,k)\Lambda_v(t,k).$$

After the frequency assignments to the selected far/near users $\{\hat{u}_k\}_{k=1}^{K}$ and $\{\hat{v}_k\}_{k=1}^{K}$ over $K$ OFDMA subchannels, the maximum supportable spectral efficiency $\mathsf{C}^{\mathrm{o}}(t)$ at time $t$ for the broadband relay-assisted multi-cellular network is given by (25) (assuming Gaussian inputs) in the presence of the orthogonal transmission protocol. In case of the simultaneous transmission protocol, the spectral efficiency $\mathsf{C}^{\mathrm{s}}(t)$ would be (assuming Gaussian inputs) expressed as in (26).

### C. Simulation Results

We consider the 7-cell network as in Fig. 5 (a main cell and 6 neighbor interfering cells) with $M = 6$, $D = 1.6$ km, $R = 1.2$ km and maximum near user range of 600 m, i.e., $D_n = 600$ m. The time-sharing coefficients are fixed across all cells as $\beta_B = 0.25$, $\beta_F = 0.25$ and $\beta_N = 0.5$. As discussed in detail in Section II, $V$ near users are served by the BS in the main cell using single-hop transmission protocols, while the co-located $U$ far users at the edge of the main cell are served by a selected RS using two-hop transmission protocols. We stated our assumptions in Sections II and IV.A on downlink transmission protocols involving orthogonal transmissions and simultaneous transmissions by the BS and RSs with varying degrees of reuse (of time and frequency resources) and characterized their spectral efficiency performance in the multi-cellular OFDMA setting under the presence of opportunistic scheduling policies in Section IV.B.

Our numerical results will consider the broadband channel model presented by [25] (and discussed in detail in [10]) for every link in the relay-based cellular network, with path loss, lognormal shadowing, frequency-selective Ricean fading with a certain power delay profile (PDP) and OFDMA-based signaling over $K = 16$ subchannels. Our specific assumptions on the channel modeling and transmission parameters are listed in Table I. In particular, we consider high-capacity line-of-sight (LOS) BS-RS links ($p = 2$ for signal links and $p = 3$ for interference links, i.e. near LOS interference), and non-LOS channels for the links from the BSs and RSs to the MSs based on the Erceg-Greenstein (EG) path loss model [26]. The lognormal shadowing standard deviation is chosen as 4 for BS-RS links and as 8 for BS-MS and RS-MS links. We also consider frequency-selective fading where each multipath fading link has four independent taps ($L = 4$) with an exponential PDP and complex Gaussian (Ricean) distribution with mean $1/\sqrt{2}$ and variance $1/2$ (i.e., Ricean $\kappa$-factor equals 1 for all taps).

In Figs. 8-9, we compare the average spectral efficiency performance of orthogonal transmission and simultaneous transmission protocols (capacity formulas provided in (25)-(26) in Section IV.B) as a function of the number of far/near users for different values of the relay reuse factor $m = 2, 6$ in the presence of the opportunistic scheduling algorithms

$$\mathsf{C}_{F,u}(t,k) = \begin{cases} \min\left[\beta_B\, C(\mathsf{SINR}_B(t,k)),\ \beta_F\,(M/m)\, C(\mathsf{SINR}_{F,u}(t,k))\right] & \text{orthogonal} \\ \min\left[\beta_B\, C(\mathsf{SINR}_B(t,k)),\ \beta_{NF}\,(M/m)\, C(\mathsf{SINR}_{F,u}(t,k))\right] & \text{simultaneous} \end{cases} \tag{24}$$

$$\mathsf{C}^{\mathrm{o}}(t) = \frac{1}{K}\sum_{k=1}^{K} \beta_N\, C(\mathsf{SINR}_{N,\hat{v}_k}(t,k)) + \min\left[\beta_B\, C(\mathsf{SINR}_B(t,k)),\ \beta_F\,\frac{M}{m}\, C(\mathsf{SINR}_{F,\hat{u}_k}(t,k))\right] \tag{25}$$

$$\mathsf{C}^{\mathrm{s}}(t) = \frac{1}{K}\sum_{k=1}^{K} \beta_{NF}\, C(\mathsf{SINR}_{N,\hat{v}_k}(t,k)) + \min\left[\beta_B\, C(\mathsf{SINR}_B(t,k)),\ \beta_{NF}\,\frac{M}{m}\, C(\mathsf{SINR}_{F,\hat{u}_k}(t,k))\right] \tag{26}$$

TABLE I
RELAY-ASSISTED MULTI-CELLULAR NETWORK LINK BUDGET FOR BS-MS, BS-RS AND RS-MS CHANNELS

| Parameter | BS-MS Channel | BS-RS Channel | RS-MS Channel |
|---|---|---|---|
| Transmit Power (RMS) (dBm) | 36 | 36 | 29 |
| Transmitter Gain (dBi) | 6 | 6 | 6 |
| Noise PSD (dBm/Hz) | -167 | -167 | -167 |
| Bandwidth (MHz) | 20 | 20 | 20 |
| Path Loss model | EG | LOS or near LOS | EG |
| Shadowing std (dB) | 8 | 4 | 8 |
| Tx antenna height (m) | 25 | 25 | 12 |
| Rx antenna height (m) | 2 | 12 | 2 |
| Carrier frequency (GHz) | 3.5 | 3.5 | 3.5 |

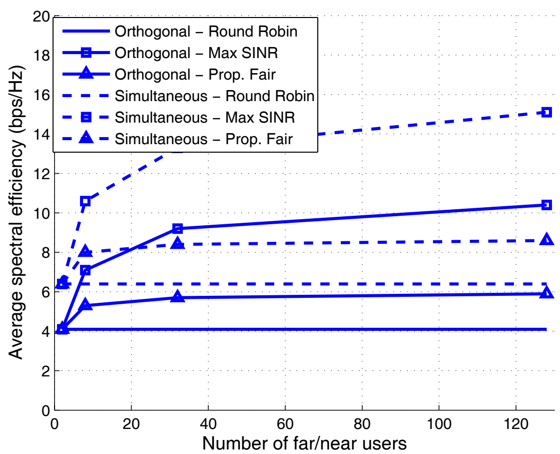


Fig. 8. Spectral efficiency comparison of orthogonal transmission and simultaneous transmission protocols for $M = 6$, $m = 6$.

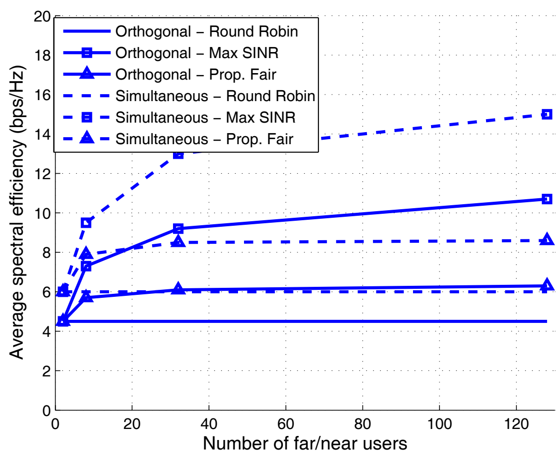


Fig. 9. Spectral efficiency comparison of orthogonal transmission and simultaneous transmission protocols for $M = 6$, $m = 2$.

discussed in Section IV.B, namely, round-robin, max-SINR and proportional-fair ($T_c = 8$) scheduling algorithms. These empirical results are obtained by averaging the expressions in (25) and (26) over a large number of randomly generated fading realizations (based on Monte Carlo simulations). The increase in average spectral efficiency with increasing number of users can be attributed to multiuser diversity realized by opportunistic scheduling of far/near users in a channel-dependent manner exploiting the time and frequency selectivity of the wireless links as well as the independence of channel variations among the far/near users.

Our simulation results clearly show that in addition to multiuser diversity benefits, further capacity enhancements are possible in cellular networks from spectrum reuse among the BS and RSs, which is fully consistent with our analytical results in Section III. More specifically, we find for the values of relay reuse factor $m = 2, 6$ that simultaneous transmission by the BS and RSs leads to a net spectral efficiency gain of 40% over orthogonal transmission. Moreover, we observe that the spectral efficiency performance is rather insensitive to $m$, leading to the conclusion that relay spectrum reuse for serving far users does not lead to a notable performance improvement. As seen in Figs. 8-9 for $m = 2, 6$, this conclusion is especially true for simultaneous transmission protocols (our simulations suggest almost identical performance for all values of $m = 1, 2, 3, 6$), while there is a very small improvement in

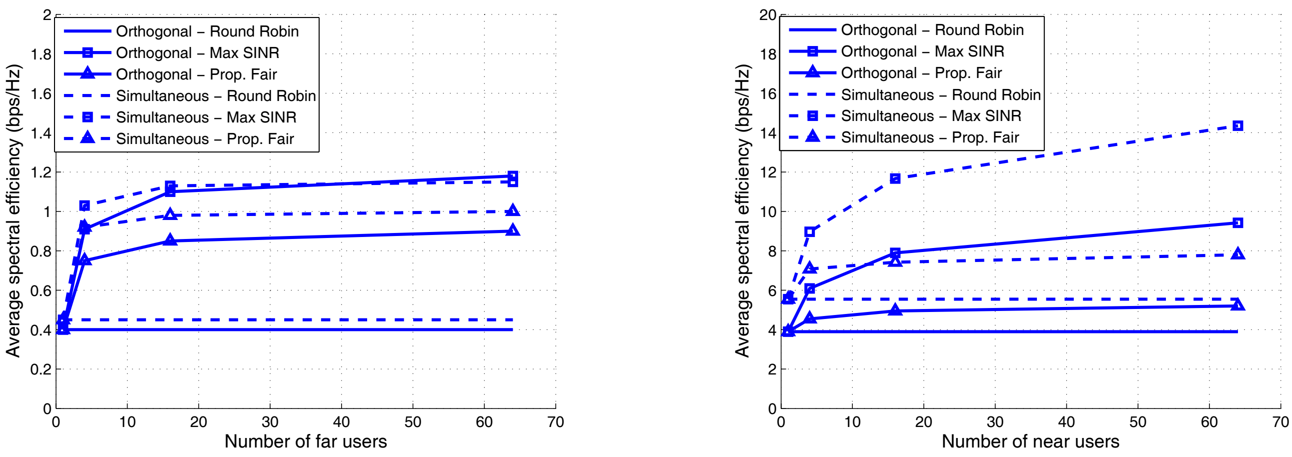


Fig. 10. Far and near user spectral efficiency comparisons of orthogonal transmission and simultaneous transmission protocols for $M = 6$, $m = 3$.

spectral efficiency with more relay spectrum reuse (i.e., as $m$ decreases) for orthogonal transmission protocols.

To gain some further understanding on the impact of spectrum reuse on the far and near users individually, we plot in Fig. 10 the average spectral efficiency performance of orthogonal transmission and simultaneous transmission protocols as a function of the number of far and near users in the case of $m = 3$ (again, the performance does not vary in a significant manner for other values of $m$, i.e., $m = 1, 2, 6$, so the plots for these cases were not included). From these results, we observe that the key beneficiaries of spectrum reuse among the BS and RSs from a spectral efficiency perspective are the near users, with the performance gains due to simultaneous transmissions by the BS and RSs (i.e., in comparison with orthogonal transmissions), allowing the near users receive from the BS over a larger portion time and frequency resources, which leads to the consequent spectral efficiency improvement. With more BS-RS spectrum reuse under simultaneous transmission protocols, it should be clear that while near users certainly achieve poorer SINR performance due to the additional interference contributed by the RSs, their performance does not get impacted in a noticeable way since they have good quality links to the BS with high signal power. In the meantime, this slight SINR loss experienced by the near users is more than compensated by the gains from simultaneous transmission protocols in terms of higher spatial reuse gains, which implies an increase in the prelog factor for the spectral efficiency expression and a resulting overall net improvement in the near user spectral efficiency. On the other hand, we observe from in Fig. 10 that the corresponding performance for the far users is not influenced significantly by BS-RS spectrum reuse. It should be noted that multihop relaying (and not spectrum reuse) is the main method to improve the QoS to the far users [1], [9], and this benefit is provided to them by both orthogonal transmission and simultaneous transmission protocols. The reason for the insensitivity of the far user spectral efficiency to the choice of the spectrum reuse policy is that while simultaneous transmission protocols again increase the spatial reuse gains (i.e., prelog factor) for the far users, they decrease the received SINR at the far users very significantly due to higher interference as a result of the additional interference contributions from the BSs, and these two opposite effects (i.e., SINR loss vs. prelog factor increase) essentially cancel out each other leading to no net gain over orthogonal transmission protocols and similar performance in both cases.

## V. Conclusions

We analyzed the spectral efficiency performance of opportunistic scheduling and spectrum reuse techniques for relay-based cellular networks in the downlink mode, based on (i) an analytical framework using tools from extreme-value theory over a simple flat-fading MBC model (isolated single-cell setting without co-channel interference) that yields accurate results even for moderately low number of users, and (ii) a system-level capacity analysis and simulation validation framework using realistic broadband multi-cellular system models in the presence of co-channel interference, frequency-selective fading and OFDMA modulation. Our study provides insights on the potential performance enhancements from multihop routing and spectrum reuse policies in the presence of multiuser diversity gains from opportunistic scheduling and helps to identify a number of key design tradeoffs associated with resource allocation and interference management in relay-based cellular networks. For instance, a key learning from our work is that significant spectral efficiency gains can be achieved through spectrum reuse by the BS and RSs in the same cell (and sector) favoring simultaneous transmission protocols over orthogonal transmission protocols, even in the absence of any kind of scheduling coordination between the BSs and RSs to control the resulting intracell interference. As a possible direction for future work, the relay-based spectrum reuse policies addressed in our work may be generalized by the incorporation of *fractional frequency reuse* (FFR) techniques [27]-[29], which would allow for dynamic QoS optimization through adaptive switching between orthogonal and simultaneous transmission protocols and adaptive selection of the relay reuse factor.

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